52. A Remark on the Theory of General Fuchsian Groups.

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Prof. M. Sugawara has recently introduced a notion of general fuchsian groups and developed a theory of automorphic functions of higher dimensions[1]. In the present note we shall show that there is another class of groups which can be treated with his method. The classical case of hyperfuchsian groups is included here as a special one (the case $m=1$ below).

§ 1. *The space* $\mathfrak{A}_{(n,m)}$. *General thetafuchsian functions in* $\mathfrak{A}_{(n,m)}$. Let us consider the set $\mathfrak{R}_{(n,m)}$ of all matrices of the type (n, m). The subset of $\mathfrak{R}_{(n,m)}$, whose elements are matrices satisfying the condition $E^{(m)}-\bar{Z}'Z>0$[2], shall be denoted by $\mathfrak{A}_{(n,m)}$[3]. Now we put $S_{(n,m)}=\begin{pmatrix} E^{(n)} & 0 \\ 0 & -E^{(m)} \end{pmatrix}$. If a matrix U of order $(n+m)$ satisfies the condition

$$(1) \qquad \bar{U}'S_{(n,m)}U=S_{(n,m)},$$

then the substitution

$$(2) \qquad W=(U_1Z+U_2)(U_3Z+U_4)^{-1}$$

carries $\mathfrak{A}_{(n,m)}$ into itself, where $U=\begin{pmatrix} U_1 & U_2 \\ U_3 & U_4 \end{pmatrix}$, and the types of U_1, U_2, U_3, U_4 are respectively (n, n), (n, m), (m, n), (m, m). Hence the matrices satisfying the condition (1) induce the displacements in the space $\mathfrak{A}_{(n,m)}$ and form a group $\Gamma_{(n,m)}$. The matrices inducing the identical displacement in $\mathfrak{A}_{(n,m)}$ are of the form $\omega E^{(n+m)}$ ($|\omega|=1$) and constitute a group $\Gamma^*_{(n,m)}$. The factor group $\Gamma_{(n,m)}/\Gamma^*_{(n,m)}$ is called the group $\mathfrak{B}_{(n,m)}$ of all displacements in $\mathfrak{A}_{(n,m)}$. $\mathfrak{B}_{(n,m)}$ is transitive in $\mathfrak{A}_{(n,m)}$: For a given point A we put

$$U_A=\begin{pmatrix} N^{-1} & -N^{-1}A \\ -M^{-1}\bar{A}' & M^{-1} \end{pmatrix}, \quad E^{(n)}-A\bar{A}'=N\bar{N}', \quad E^{(m)}-\bar{A}'A=M\bar{M}'.$$

Then U_A carries A into the zero point and $U_A \in \Gamma_{(n,m)}$.

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1) M. Sugawara, Über eine allgemeine Theorie der Fuchsschen Gruppen und Theta-Reihen, Ann. Math. **41**, 488–494; M. Sugawara, On the general Zetafuchsian functions, Proc. **16** (1940), 367–372; M. Sugawara, A generalization of Poincaré-space, Proc. **16** (1940), 373–377. In the sequel these papers will be cited as S. I, S. II, S. III respectively.

2) By $E^{(m)}$ we mean the unite matrix of order m. $H>0$ means that a hermitian matrix H is positive definite. The same notations as in S. I will be used in this note.

3) If we define the distance between two points Z_1 and Z_2 as $[Sp\overline{(Z_1-Z_2)}'(Z_1-Z_2)]^{\frac{1}{2}}$ then $\mathfrak{A}_{(n,m)}$ is an open, bounded, convex set in a complete metric space $\mathfrak{R}_{(n,m)}$.