29. On the Behaviour of an Inverse Function of a Meromorphic Function at its Transcendental Singular Point, III.

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1. Nevanlinna's fundamental theorems.

Let $w=w(z)=f(z)$ be a meromorphic function for $|z|<\infty$ and $z=\varphi(w)$ be its inverse function. Let K be the Riemann sphere of diameter 1, which touches the w-plane at $w=0$ and $[a, b]=\dfrac{|a-b|}{\sqrt{(1+|a|^2)(1+|b|^2)}}$.

A δ-neighbourhood U of w_0 is the connected part of the Riemann surface F of $\varphi(w)$, which lies in $[w, w_0]<\delta$ and has w_0 as an inner point or as a boundary point. Let U correspond to Δ on the z-plane, then $[f(z), w_0]<\delta$ in Δ and $[f(z), w_0]=\delta$ on the boundary of Δ. We assume that Δ extends to infinity. Let z_0 be a point on the z-plane and Δ_r, θ_r be the common part of Δ and $|z-z_0|\leqq r$ and $|z-z_0|=r$ respectively. We put $A(r, w; \Delta)=$ the area on K, which is covered by $w=f(z)$, when z varies in Δ_r, $S(r, w; \Delta)=\dfrac{A(r, w; \Delta)}{\pi\delta^2}$, where $\pi\delta^2$ is the area of $[w, w_0]\leqq\delta$ on K, $n(r, a, w; \Delta)=$ the number of zero points of $f(z)-a$ in Δ_r, where $[a, w_0]<\delta$.

$$N(r, a, w; \Delta)=\int_{r_0}^{r}\frac{n(r, a, w; \Delta)}{r}\,dr\,,$$

$$m(r, a, w; \Delta)=\frac{1}{2\pi}\int_{\theta_r}\log\frac{1}{[w(re^{i\varphi}), a]}\,d\varphi\,,$$

$$T(r, a, w; \Delta)=N(r, a, w; \Delta)+m(r, a, w; \Delta)\,,$$

$L(r)=$ the total length of the curve on K, which corresponds to θ_r. Then we have the following theorem[1], which corresponds to *Nevanlinna's first fundamental theorem.*

Theorem I. $\quad T(r, a, w; \Delta)=T(r, w; \Delta)+O\Big(\int_{r_0}^{r}\frac{L(r)}{r}\,dr\Big)\,,$

where $$T(r, w; \Delta)=\int_{r_0}^{r}\frac{S(r, w; \Delta)}{r}\,dr\,.$$

We will call $T(r, w; \Delta)$ the characteristic function of $f(z)$ in Δ and

1) C. f. K. Kunugui: Une généralisation des théorèmes de MM. Picard-Nevanlinna sur les fonctions méromorphes. Proc. **17** (1941), 283–289.

Y. Tumura: Sur le problème de M. Kunugui. Proc. **17** (1941), 289–295.

Mr. Tumura obtained the same result as Theorem 1, but he informed me that he found a mistake in his proof and will publish a revised proof in this proceedings.