39. On Krull's Conjecture Concerning Completely Integrally Closed Integrity Domains. I.

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In the important papers, *Allgemeine Bewertungstheorie*, Crelles Journal 167 (1932) and *Beiträge zur Arithmetik kommutativer Integritätsbereiche*, Math. Zeitschr. 41 (1936), W. Krull gave a conjecture[1] that every completely integrally closed (= vollständig ganz-abgeschlossen)[2] integrity domain can always be expressed, in its quotient field, as an intersection of special valuation rings[3]. On ignoring addition A. H. Clifford has worked on the problem whether or not every Archimedean partially ordered abelian group can be embedded in a real component vector group, or what is the same, represented faithfully by (finite) real-valued functions[4]. In the following we want to show that the conjectures can not be the case in general. We shall first take up the simpler case of partially ordered abelian groups; The case of integrity domains will be treated in Part II.

Now, let A be a complete Boolean algebra and $\Omega=\Omega(A)$ be its representation space, that is, the totality of prime dual ideals of A with Stone-Wallman's topology; when $a \in A$ the so-called a-set, the set of prime dual ideals containing a, is an open and closed subset of Ω, and conversely every open and closed subset of Ω is an a-set; the system of all the a-sets forms a basis of closed sets in $\Omega : \Omega$ is thus a totally disconnected bicompact T_1-space. In Ω Borel sets coincide with open and closed sets (a-sets) *mod.* sets of first category. From this follows, as T. Ogasawara pointed out recently[5], that in Ω every Borel-measurable function finite except on a set of first category coincides except on a set of first category with a (real and $\pm\infty$-valued) continuous function finite except on a nowhere dense set, and the totality of the functions of the last class, namely (real and $\pm\infty$-valued) continuous functions on Ω finite except on nowhere dense sets, forms a vector-lattice $\mathfrak{L}_\Omega=\mathfrak{L}_{\Omega(A)}$. The order relation in $\mathfrak{L}_\Omega$ is point-wise as usual. As for addition it is as follows: the sum $g+h$ of two elements g, h in $\mathfrak{L}_\Omega$ is the continuous function on Ω finite except on a nowhere dense set coinciding with the

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1) § 4 and Part II, § 1, respectively, of the cited papers by W. Krull. Cf. also P. Lorenzen, Abstrakte Begründung der multiplikativen Idealtheorie, Math. Zeitschr. **45** (1939).

2) An integrity domain I is called completely integrally closed when an element x in its quotient field such that $x^n a \in I$ $(n=1, 2, \dots)$ for a suitable element $a\ (\neq 0)$ in I lies necessarily in I.

3) An (exponential) valuation is called special when its value group consists of real numbers.

4) A. H. Clifford, Partially ordered abelian groups, Ann. Math. **41** (1940).

5) T. Ogasawara, On Boolean spaces (in Japanese), Zenkoku-Sizyo-Sugaku-Danwakai **230** (1941).