# 49. *On Krull's Conjecture Concerning Completely Integrally Closed Integrity Domains, II.*

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The case of partially ordered abelian groups being settled in Part I[1], let us turn to integrity domains; we want to obtain an integrity domain which is completely integrally closed but can never be expressed as an intersection of special valuation rings[2]. Our following construction depends however on that of Part I.

Let $A$ be a complete Boolean algebra satisfying the condition in Part I, Lemma 1; there be a countable set of non-atomic non-zero elements $v_i$ in $A$ so that for any $a>0$ in $A$ we have $a \geqq v_i$ for a suitable $i$[3]. Denote its representation space by $\Omega=\Omega(A)$. Then the lattice-ordered abelian group $L_\Omega$ of continuous functions on $\Omega$, taking (rational) integers and $\pm\infty$ as values and finite except on nowhere dense sets, cannot, as was shown in Part I, be represented faithfully by (finite) real-valued functions (over any space). Now, let $K$ be a field, and consider, abstractly, variables $x(\mathfrak{p})$ which are in one-one correspondence with the points $\mathfrak{p}$ in $\Omega$. When $\{\mathfrak{p}_1, \mathfrak{p}_2, \dots, \mathfrak{p}_s\}$ is a finite set of (distinct) points of $\Omega$, a polynomial of the variables $x(\mathfrak{p}_1), x(\mathfrak{p}_2), \dots, x(\mathfrak{p}_s)$ over $K$ will be called in the following a $\mathfrak{p}_1\mathfrak{p}_2\cdots\mathfrak{p}_s$-polynomial. Let $\{\mathfrak{p}_1, \mathfrak{p}_2, \dots, \mathfrak{p}_t\}$ be a subsystem of $\{\mathfrak{p}_1, \mathfrak{p}_2, \dots, \mathfrak{p}_s\}$. A $\mathfrak{p}_1\mathfrak{p}_2\cdots\mathfrak{p}_s$-polynomial $F(\mathfrak{p}_1\cdots\mathfrak{p}_s)\ \Big(=F\big(x(\mathfrak{p}_1), \dots, x(\mathfrak{p}_s)\big)\Big)$ is said to be reduced to a $\mathfrak{p}_1\cdots\mathfrak{p}_t$-polynomial $F(\mathfrak{p}_1\cdots\mathfrak{p}_t)$, when it becomes the latter by putting $x(\mathfrak{p}_{t+1})=\cdots=x(\mathfrak{p}_s)=1$; in symbol $F(\mathfrak{p}_1\cdots\mathfrak{p}_s)\to F(\mathfrak{p}_1\cdots\mathfrak{p}_t)$. Further, let $P$ be a set of first category in $\Omega$ and suppose that for each finite system $\{\mathfrak{p}_1, \dots, \mathfrak{p}_s\}$ of points in $\Omega$ not belonging to $P$ there is given a $\mathfrak{p}_1\cdots\mathfrak{p}_s$-polynomial $F(\mathfrak{p}_1\cdots\mathfrak{p}_s)$. If here $F(\mathfrak{p}\cdots\mathfrak{p}_s)\to F(\mathfrak{p}_1\cdots\mathfrak{p}_t)$ whenever $\{\mathfrak{p}_1, \dots, \mathfrak{p}_s\}\supset\{\mathfrak{p}_1, \dots, \mathfrak{p}_t\}$, we call this whole scheme a *polynomial series* on $\Omega$ and denote it by $\{F; P\}=\{F(\mathfrak{p}\cdots\mathfrak{p}); P\}$. Two polynomial series $\{F; P\}$ and $\{F'; P'\}$, such that $F(\mathfrak{p}_1\cdots\mathfrak{p}_s)=F'(\mathfrak{p}_1\cdots\mathfrak{p}_s)$ for every $\{\mathfrak{p}_1, \dots, \mathfrak{p}_s\}\subset\Omega-Q$, where $Q$ is a set of first category containing $P, P'$, will be called equivalent; we consider equivalent polynomial series as one and the same. The sum (product) of two polynomial series $\{F_1; P_1\}$ and $\{F_2; P_2\}$ is defined by taking $F_1(\mathfrak{p}_1\cdots\mathfrak{p}_s)+F_2(\mathfrak{p}_1\cdots\mathfrak{p}_s)$ $\big(F_1(\mathfrak{p}_1\cdots\mathfrak{p}_s)F_2(\mathfrak{p}_1\cdots\mathfrak{p}_s)\big)$ for $\{\mathfrak{p}_1, \dots, \mathfrak{p}_s\}\subset\Omega-(P_1\smile P_2)$. Then the totality of polynomial series (the totality of classes of equivalent polynomial series, to be exact) forms a ring $R_\Omega$,

1) T. Nakayama, On Krull's conjecture concerning completely integrally closed integrity domains, I., Proc. **18** (1942), 185.

2) See the papers cited in Part I. Cf. also Enzyklopädie der Math. Wiss. $I_1$, 11, p. 40.

3) For instance, let $A$ be the complete Boolean algebra of regular open sets of the interval (0, 1).