PAPERS COMMUNICATED

45. *On the Behaviour of a Meromorphic Function in the Neighbourhood of a Closed Set of Capacity Zero.*

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1. *Nevanlinna's fundamental theorems.*

Let E be a bounded closed set of capacity 0 on the z-plane, which is contained in a bounded domain D and $w=w(z)=f(z)$ be meromorphic in $D-E$ and have every point of E as an essential singularity. Since E is of capacity 0, by Evans' theorem[1], we can distribute a positive mass $d\mu(a)$ on E, such that

$$u(z)=\int_E \log\frac{1}{|z-a|}d\mu(a)\,,\qquad \int_E d\mu(a)=1\,, \tag{1}$$

is harmonic in $D-E$ and $u(z)=\infty$ at every point of E. Let $\theta(z)$ be the conjugate harmonic function of $u(z)$ and put

$$t=e^{u(z)+i\theta(z)}=r(z)e^{i\theta(z)}\,. \tag{2}$$

This $r(z)$ plays the similar rôle as $|z|$ in the theory of meromorphic functions for $|z|<\infty$. Let C_r be the niveau curve: $r(z)=\text{const.}=r$, then C_r consists of finite number of closed curves surrounding E. We remark that $\int_{C_r}d\theta(z)=\int_{C_r}\frac{\partial u}{\partial n}ds=2\pi\int_E d\mu=2\pi$, where n is the inner normal of C_r. We assume that D is bounded by an analytic Jordan curve C and the domain bounded by C and C_r be denoted by Δ_r. Let K be the Riemann sphere of diameter 1, which touches the w-plane at $w=0$ and put $[a,b]=\frac{|a-b|}{\sqrt{(1+|a|^2)(1+|b|^2)}}$, $n(r,a)=$the number of zero points of $f(x)-a$ in Δ_r,

$$N(r,a)=\int_{r_0}^{r}\frac{n(r,a)}{r}dr\,,$$

$$m(r,a)=\frac{1}{2\pi}\int_{C_r}\log\frac{1}{[w(z),a]}d\theta(z)\,,$$

$$T(r,a)=m(r,a)+N(r,a)\,,$$

$A(r)=$the area on K, which is covered by $w=f(z)$, when z varies in Δ_r and $S(r)=\frac{A(r)}{\pi}$.

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1) Evans: Potentials and positively infinite singularities of harmonic functions. Monathefte f. Math. u. Phys. **43** (1936).