# 88. On the Cauchy's Integral Theorem.

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S. Pollard have obtained a following theorem, for extension of the Cauchy's well-known theorem[1]:

Theorem. *Let $C$ be any closed plane jordan curve with no multiple points, and let $D$ be a connected domain enclosed by $C$ in its interior. Let further $f(z)$ be a uniform function defined in $D$ and satisfy the following conditions:*

$1^\circ$ *The real and imaginary parts of $f(z)$ have partial derivative which satisfy Cauchy's equations at all points within $D$, and are integrable over every rectangle within $D$... integrability being understood either in the sense of Riemann, or more general sense of Lebesgue.*

$2^\circ$ *$f(z)$ is continuous on $C$ so far as values at points within and on it are concerned.*

$3^\circ$ *$C$ is a curve of bounded variation.*

*Then the integral of $f(z)$ round the contour $C$ is zero, that is*

$$\int_C f(z)\,dz = 0$$

But the proof of this theorem given by S. Pollard[1] seems to us to be insufficient for the general case[2]. The object of this paper is to give a correct proof of this theorem which modifies and simplifies Pollard's proof.

First, let us give certain lemmas.

Lemma 1. *Suppose that $C$ be a rectifiable plane curve with no multiple point and denote its length by $L$. Then, for any positive number $\varepsilon$, there exists a polygon $\pi$ inside $C$, which satisfies the following conditions:*

$(1^\circ)$ *Its sides are parallel to one or other of the axes.*

$(2^\circ)$ *It is possible to divide $C$ and $\pi$, into equal number $n$ of small arcs $C_1, C_2, \dots, C_n$ and broken lines $\pi_1, \pi_2, \dots, \pi_n$ respectively, so that, for each pair $(C_i, \pi_i)$ $(i=1, 2, \dots, n)$, hold the inequality*

$$\rho(a, b) < \varepsilon \quad as \quad a \in C_i,\ b \in \pi_i$$

*and that $n\varepsilon < 4L$.*

$(3^\circ)$ *Denoting by $l(\pi)$ the length of $\pi$, we have $l(\pi) < 11L$: $l(\pi)$ is therefore uniformly bounded.*

1) S. Pollard: On the conditions for Cauchy's theorem, proceedings of the London Math. Soc. Second Series, vol. **21** (1923), p. 456–482. Cf. also, E. Kamke: Zu dem Integralsatz von Cauchy, Math. Zeitschrift, Bd. **35** (1932), p. 535–543; J.L. Walsh: Approximation by polynomials in the complex domain, Paris, 1935, p. 9.

2) For example, consider the case where $C$ has an angular point with angle wich is sufficiently small, and one of the tangents at this point is parallel to one or other of the axes. In this case, their non-consecutive links surely overlap, and the chain is not "regular."