# 119. On the Newtonian Capacity and the Linear Measure.

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I. Given a bounded set $E$ of points in Euclidean plane $\omega$, let us denote the diameter of $E$, as usual, by $\delta(E)$. We shall denote, for each $\varepsilon > 0$, by $\wedge_\varepsilon(E)$ the lower bound of all the sums $\sum_i \delta(E_i)$ where $\{E_i\}_{i=1,2,\dots}$ is an arbitrary partition into a sequence of sets that have diameters less than $\varepsilon$ and no two of which have common points. Making $\varepsilon$ approach to zero, the number $\wedge_\varepsilon$ tends, in a monotone non-decreasing way, to a unique limit (finite or infinite) which is called the linear measure of $E$ and will be denoted by $\wedge(E)$.

It is known that $\wedge(E)$ has the property of Carathéodory's outer measure[1] and therefore all the Borel sets are measurable in the sense of linear measure and $\wedge(E)$ is an additive function of linearly measurable set.

We shall say that $\mu$ is a positive distribution of the mass $m$ on the Borel set $E$, if $\mu$ is a non-negative and completely additive set function defined for all the Borel sets in $\omega$ such that $\mu(E)=m$ and $\mu(\omega-E)=0$.

Given a fixed point $P$, a variable point $Q$, let us denote the distance from $P$ to $Q$ by $r_{PQ}$. For an arbitrary distribution of positive mass on the set $E$, the Lebesgue-Stieltjes integral

$$u(p)=\int_E \frac{d\mu(Q)}{r_{PQ}}$$

represents a function ($\leqq +\infty$) of point $P$ which we call the Newtonian potential due to the distribution $\mu$.

For every distribution of the *unit* mass on the set $E$, the potential $u(p)$, $p\varepsilon E$, has a positive upper bound (finite or infinite). Denoting by $V(E)$ the lower bound of this uppor bound, for all possible distributions, we call

$$C(E)=\frac{1}{V(E)}$$

the newtonian capacity of the set $E$.

As is known, the Newtonian capacity $C(E)$ is not necessarily addtive even in the restrictive sense.

II. Mr. Frostman has proved in his thesis[2] the following theorem.

Theorem I. If the set $E$ is of linear measure zero, the Newtonian capacity of $E$ is zero.

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1) F. Hausdorff: Dimension und äusseres Mass, Math. ann. Vol. **79** (1919) pp. 157–179.

2) Frostman: Potentiel d'équilibre et capacite des énsemble. Lund (1935) p. 89 Mr. Frostman has proved the theorem concerning more general measure and capacity.