13. *On the Cluster Set of a Meromorphic Function.*

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1. Let Δ be a bounded domain on the z-plane and z_0 be a non-isolated accessible boundary point on the boundary Γ of Δ. We denote the part of Δ, Γ in $|z-z_0| \leqq r$ by Δ_r, Γ_r respectively and the part of $|z-z_0|=r$, which lies in Δ by θ_r. Let $w=f(z)$ be one-valued and meromorphic in Δ and W_r be the set of values taken by $f(z)$ in Δ_r and $\overline{W}_r$ be its closure. Then

$$\lim_{r\to 0} \overline{W}_r = H_\Delta(z_0) \tag{1}$$

is called the cluster set of $f(z)$ in Δ at z_0.

Let $\zeta(\neq z_0) \in \Gamma$ and $H_\Delta(\zeta)$ be the cluster set of $f(z)$ at ζ and

$$V_r(\Gamma) = \sum H_\Delta(\zeta), \quad \text{added for all } \zeta(\neq z_0) \text{ on } \Gamma_r \tag{2}$$

and $\overline{V}_r(\Gamma)$ be its closure. Then

$$\lim_{r\to 0} \overline{V}_r(\Gamma) = H_\Gamma(z_0) \tag{3}$$

is called the cluster set of $f(z)$ on Γ at z_0.

It is obvious that $H_\Delta(z_0) \supset H_\Gamma(z_0)$. Iversen[1] proved that *every boundary point of $H_\Delta(z_0)$ belongs to $H_\Gamma(z_0)$.*

Let $\zeta \in \Gamma$. If for any $\varepsilon > 0$, there exists a neighbourhood U of ζ, such that $|f(z)| \leqq m+\varepsilon$ in U, then we will write: $|f(\zeta)| \leqq m$. Then as an immediate consequence of the Iversen's theorem, we have[2]: *Let $f(z)$ be regular and bounded in Δ. If $\overline{\lim_{z\to z_0}} |f(z)| \leqq m$, when z tends to z_0 on Γ, then $\overline{\lim_{z\to z_0}} |f(z)| \leqq m$, when z tends to z_0 in Δ.*

I will here extend the Iversen's theorem in the following way.

Let E be a closed set of capacity zero on Γ and $z_0 \in E$ and $U(\Gamma - E) \neq 0$ for any neighbourhood U of z_0. We denote the part of E in $|z-z_0| \leqq r$ by E_r. Let

$$V_r(\Gamma - E) = \sum H_\Delta(\zeta), \quad \text{added for all } \zeta(\neq z_0) \text{ on } \Gamma_r - E_r \tag{4}$$

and $\overline{V}_r(\Gamma - E)$ be its closure. Then

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K. Noshiro: On the theory of cluster sets of the analytic functions. Jour. Fac. Sci. Hokkaido Imp. Univ. **6** (1938),—On the singularities of analytic functions. Jap. Jour. Math. **17** (1940).

2) K. Kunugui, l. c.