31. On the Semi-ordered Ring and its Application to the Spectral Theorem. II.

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In the first place, our note[1] "On the semi-ordered ring and its application to the spectral theorem" contained, in its proof of algebraic part, a falsy argument, which we shall correct here. Namely, its lemma in § 1 (p. 557) was incorrect; it ought to have referred only to a normal subgroup generated by positive elements. The following revised proof runs more or less in the same line as Vernikoff-Krein-Tovbin's,[2] but we may put emphasis on that neither associativity (nor commutativity) nor ring property is used; we simply deal with abelian groups with operators. Indeed, as an application of such mode of our approach, we can determine the structure of the additive group of *bounded automorphisms* of a semi-ordered abelian group (satisfying certain conditions); this forming the second purpose of the present supplementary note.

Let G be a semi-ordered abelian group with real multipliers[3], such that[4]

(i) if $x \geqq 0$ and $y \geqq 0$ then $x+y \geqq 0$,

(ii) if $x \geqq 0$ and $-x \geqq 0$ then $x=0$,

(iii) if $x \geqq 0$ and α (real number) $\geqq 0$ then $\alpha x=0$.

Let G possess further an operator domain $\Omega=\{A\}$ which is by itself a semi-ordered abelian group (in the same sense as G) such as

(vii) if $x \geqq 0$ (in G), $A \geqq 0$ (in Ω) then $Ax \geqq 0$ (in G),

(viii) $(A+B)x=Ax+Bx$, $A(x+y)=Ax+Ay$, $A(\alpha x)=\alpha Ax$,

and let moreover

(ix) Ω possess an Archimedean unit I which satisfies $Ix=x$, $x \in G$.

Then we have

Lemma. Every normal subgroup of G generated by a certain system of positive elements is always allowable with respect to Ω.

For, our former proof remains valid certainly in this case.

Suppose now

(iv) G itself possess an Archimedean unit e,

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1) Proc. **18** (1942), 555.
2) Sur les anneaux semi-ordonnés, C. R. URSS, **30** (1941), p. 758.
3) Cf. a remark below
4) The numbers for the conditions are in accordahce with our former note.