96. *Bohr Compactifications of a Locally Compact Abelian Group I.*

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§ 1. *Introduction.* The purpose of this paper[1] is to give a general theory of Bohr compactifications of a locally compact abelian group. We begin with the discussion of general properties of Bohr compactifications (§ 2). The theory of a monothetic or a solenoidal compact group, which recently became more important because of its newly discovered relations[2] with the theory of ergodic measure preserving transformations or flows[3] having a pure point spectrum, will then be discussed as a special case of our theory (§§ 5, 6). We shall give a new proof to a theorem of A. Weil[4] to the effect that a non-discrete monothetic group is compact whenever it is locally compact (Theorem 5). Among other things it will then be shown that a compact abelian group, whose cardinal number does not exceed 2^c, is a solenoidal group if and only if it is connected (Theorem 6). The existence of a non-separable[5] monothetic or a solenoidal group[6] which once seemed to be rather surprising will now turn out to be quite a natural fact after a general method of taking Bohr compactifications of a locally compact abelian group is obtained (§ 3 and Theorems 7,15). The more detailed investigations of this phenomenon were published in a paper[7] of one of the present authors in which the problems concerning cardinal numbers of a compact abelian group are discussed.

Further, the structure of the universal Bohr compactification of an arbitrary locally compact abelian group, and also the structure of the universal monothetic or the universal solenoidal compact group will be determined by means of the method of character groups (§ 4, 5, 6). This result is closely related with the results obtained in another occasion[8] in connection with the problems of the normed ring of a locally compact abelian group. Finally it is interesting to com-

1) This paper is divided into two parts: I (§§ 1, 2, 3, 4) and II (§§ 5, 6).

2) P. R. Halmos and J. von Neumann, Annals of Math., **43** (1942).

3) flow=one parameter group of measure preserving transformations.

4) A. Weil: L'intégration dans les groupes et leurs applications, Actualités, Paris, 1939.

5) A topological group (or space) is called separable if it satisfies the second countability axiom of Hausdorff.

6) The infinite direct sum $\sum_a \oplus K_a$ of a continuum number of compact abelian groups K_a, each of which is topologically isomorphic with the additive group of real numbers mod. 1 with the usual compact topology, gives an example of a monothetic (or solenoidal) compact group which is not separable.

7) S. Kakutani, On cardinal numbers related with a compact abelian group, Proc., **19** (1943), 366–372.

8) K. Kodaira and S. Kakutani, Normed ring of a locally compact abelian group, Proc. **19** (1943), 360–365.