95. Note on Free Topological Groups.

By Tadasi NAKAYAMA.
Department of Mathematics, Nagoya Imperial University.
(Comm. by T. TAKAGI, M.I.A., Oct. 12, 1943.)

The notion of free topological groups has been introduced by A. Markoff[1]. The present note is to give two remarks concerning it. The first is to show that free topological groups are always maximally almost periodic; the case of discrete free groups being well-known[2]. And this fact, combined with an additional observation, shows us the embeddability of any completely regular space into a totally bounded topological group as a closed subspace.

Our second remark is concerned with a refinement of the notion of free topological groups, namely, that of uniform free topological groups generated by a uniform space. It contains Markoff's free group as its special case where the completely regular space is considered under its finest uniformity.

1. *Maximally almost periodicity of free topological groups.* Let R be a completely regular space. The free topological group F generated by R is characterized by the properties:

i) R is a subspace of F,

ii) R generates F algebraically,

iii) Given a continuous mapping φ of R into any topological group, there exists a continuous homorphism Φ of F into G which is an extension of the mapping φ.

Theorem 1. The free topological group F is always maximally almost periodic.

Proof. Let g be an element of F different from the unit 1. With a certain number, say n, of elements $u_1, u_2, \dots, u_n$ from R, g is expressed in a from

$$g = u_{i_1}^{\varepsilon_1} u_{i_2}^{\varepsilon_2} \cdots u_{i_m}^{\varepsilon_m} \qquad (\varepsilon_k = \pm 1).$$

Consider then the (algebraic, discrete) free group F_0 generated by the n elements $u_1, u_2, \dots, u_n$. There exists[3] in F_0 an invariant subgroup N_0 of a finite index and not containing g. Let $A(h)$ $(h \in F_0)$ be a faithful unitary representation of the finite factor group F_0/N_0, and put for the sake of simplicity

$$A_1 = A(u_1),\ A_2 = A(u_2),\ \dots,\ A_n = A(u_n).$$

Since the group[4] $\mathfrak{U}$ of unitary matrices, of the same degree as the representation $A(h)$, is connected, there exist in $\mathfrak{U}$ n continuous paths π_i

1) A. Markoff, On free topological groups, C. R. URSS. **31** (1941).

2) J. v. Neumann-E. P. Wigner, Minimally almost periodic groups, Ann. Math. **41** (1940); V. L. Nismewitsch, Über Gruppen, die durch Matrizen über einem kommutativen Feld isomorph darstellbar sind, Rec. Math. **51** (1940); K. Iwasawa, Iso-Sugaku **4** (1942).

3) See 2).

4) Topologized as usual.