91. Conformal and Concircular Geometries in Einstein Spaces.

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S. Sasaki[1] has recently studied the spaces with normal conformal connexions whose groups of holonomy fix a point or a hypersphere, and derived the fundamental theorem: If the group of holonomy of a space C_n with a normal conformal connexion is a subgroup of the Möbius group which fixes a point (or a hypersphere), the C_n is a space with a normal conformal connexion corresponding to the class of Riemann spaces conformal to each other including an Einstein space with a vanishing (or non-vanishing) scalar curvature. The converse is also true.

But, it seems to me that, the group of holonomy of C_n fixing a point or a hypersphere, the whole space C_n is not necessarily conformal to an Einstein space, but it may admit of an exceptional point or hypersurface. The first purpose of this Note is to study such exceptional cases.

S. Sasaki[1] has also studied the spaces with normal conformal connexions whose groups of holonomy fix two points or hyperspheres. These spaces are closely related to the Einstein spaces which admit a concircular transformation[2]. The second purpose of this Note is to consider the relations between the conformal and the concircular geometries in these spaces.

§ 1. *Spaces whose groups of holonomy fix a point or a hypersphere.*

Let us consider a space C_n with a normal conformal connexion and take the Veblen repere $[A_0, A_\lambda, A_\infty]$[3] in each tangent space, then, the normal conformal connexion may be expressed by the following formulae:

$$(1.1)\qquad \begin{cases} dA_0 = \qquad\qquad dx^\lambda A_\lambda\,, \\ dA_\mu = \Pi^0_{\mu\nu}dx^\nu A_0 + \Pi^\lambda_{\mu\nu}dx^\nu A_\lambda + \Pi^\infty_{\mu\nu}dx^\nu A_\infty\,, \\ dA_\infty = \qquad\qquad \Pi^\lambda_{\infty\nu}dx^\nu A_\lambda\,, \end{cases}$$

where

1) S. Sasaki: On the spaces with normal conformal connexions whose groups of holonomy fix a point or a hypersphere, I. Japanese Journal of Mathematics, vol. **18** (1943), pp. 615–622, II. ibidem, pp. 623–633. These papers will be cited as S. I. and S. II. respectively.

2) K. Yano: Concircular geometry I. Concircular transformations, Proc. **16** (1940), 195–200; II. Integrability conditions of $\rho_{\mu\nu}=\phi g_{\mu\nu}$, ibidem, pp. 354–360; III. Theory of curves, ibidem, pp. 442–448; IV. Theory of subspaces, ibidem, pp. 505–511; V. Einstein spaces, ibidem, **18** (1942), pp. 446–451.

3) K. Yano: Sur la théorie des espaces à connexion conforme, Journal of the Faculty of Science, Imperial University of Tokyo, vol. 4, part 1 (1939), pp. 1–59.

The greek indices run from 1 to n and the latin ones from $\dot{1}$ to $\dot{n}-\dot{1}$.