90. The Exceptional Values of Functions with the Set of Linear Measure Zero of Essential Singularities, II.

By Syunzi KAMETANI.

Tokyo Zyosi Koto-Sihan-Gakko, Koisikawa, Tokyo.

(Comm. by S. KAKEYA, M.I.A., Oct. 12, 1943.)

1. Let D be a domain and E be a compact sub-set, of D, of linear measure zero in the sense of Carathéodory.

If $w=f(z)$ is regular in $D-E$ and has E as its essential singularities, then, *near each point* $z_0 \in E$, $f(z)$ *takes every finite value except perhaps those belonging to a set of Newtonian capacity Zero.* This result, an extension of the one obtained by M. L. Cartwright[1] was proved in our former Note with the same title as the present one, Proc. **17** (1941).

Now, according to the result obtained recently by the present author,[2] a set of Newtonian capacity zero may be the sum of enumerably infinite sets of Carathéodory's linear measure finite, so that the exceptional set stated above might be of linear measure positive.

In this note, we shall show that the intersection of the exceptional set with any straight line is of linear measure zero.

2. We shall denote by $m(E)$ Carathéodory's lineare measure[3] or the length of E, by E^e the complementary set of E, and by $\{p\,;\,P\}$ the set of all the points p with the property P.

Lemma 1. *Let F be a closed set on a rectifiable Jordan arc L[4] and of positive linear measure. Then there exists a point $\zeta_0 \in F$ such that*

$$\int_{F\cdot A} d\zeta \neq 0$$

for every sufficiently small arc $A(\subset L)$ containing ζ_0.

Proof. Let L be represented by the equation:

$$\zeta=\zeta(s) \qquad (0 \leqq s \leqq l)$$

with the arc length s as its parameter.

Then, at almost every point of s, $\zeta'(s)$ exists and $|\zeta'(s)|=1$.

Writing $\zeta'(s)=e^{i\varphi(s)}$, $\big(\varphi(s)\,;\text{ real}\big)$, we have

$$\int_{F\cdot A} d\zeta = \int_{M\cdot I} e^{i\varphi(s)} ds\,,$$

1) M. L. Cartwright. The exceptional values of functions with a non-enumerable set of essential singularities, Quart. J. Math., Vol. **8** (1937).

2) S. Kametani. On some properties of Hausdorff's measure and the concept of capacity in generalized potentials, Proc. **18** (1942).

3) S. Saks. Theory of the Integral (1937), p. 53.

4) We may suppose that the set F does not contain any of the end points of L. We suppose it hereafter, if necessary, without explicitly saying so.