133. On the Phenomena of Instability in Undamped Quasi-harmonic Vibration. Part I.

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The quasi-harmonic vibration, namely, the vibration of a system with periodically varying elasticity or damping coefficient or inertia mass, is present in such widely different kinds of engineering problems as, for example, a two-pole turbo-generator, a condenser microphone, an electric locomotive of the side-rod type, a two-blade propeller, an internal combustion engine with cranks and pistons, etc. It is possible to show that the equation of undamped quasi-harmonic vibration of any case is generally involved in the expression of the type:

$$\frac{d}{dt}\left\{P(t)\frac{dX}{dt}\right\}+Q(t)X=0\,, \tag{1}$$

where $P(t)$, $Q(t)$ are periodic functions of time. For meeting every practical need, it is advisable to decompose equation (1) to three simple cases, namely

$$P(t)=\text{const.},\qquad Q(t)=Q_0+Q_1\cos 2pt\,, \tag{1 a}$$

$$P(t)=\text{const.},\qquad Q(t)=1/R(t)=1/(R_0+R_1\cos 2pt)\,, \tag{1 b}$$

$$Q(t)=\text{const.},\qquad P(t)=P_0+P_1\cos 2pt\,, \tag{1 c}$$

where $2p$ is the frequency of periodic variation of such coefficient as elasticity or damping or inertia mass.

Case (1 a), having already called attention of many investigators, is well known to be solved with Mathieu's functions, whereas cases (1 b), (1 c) are not treated as simple as in case (1 a). If however R_1, P_1 were small quantities, their solutions would be represented in the forms of expansion in series or in other approximate ones. Since with such restriction as R_1, P_1 being small, the problem is liable to be outside the theoretical interest and also to be remote from practical use, it is now of pressing importance to obtain more satisfactory solutions that should be adapted to any value of R_1 or P_1.

Upon examining the nature of the equations, it has been found that transformation of certain variables aids us to formulate such solutions as will answer, at least, to some cases of ripple in periodically varying coefficient, as a result of which it is possible for the restriction of R_1, P_1 just mentioned to be precluded.

The expression (1 b) can also be written

$$\frac{d^2X}{d\tau^2}+\frac{\omega_{0p}^2}{1-k^2\sin^2\tau}X=0\,, \tag{2}$$

where