16. *On Group Rings of Topological Groups.*

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§ 1. Let G be a locally compact topological group, satisfying the second axiom of countability and μ a left invariant Haar measure on G. We denote as usual by $L^p(G)$ $(p \geqq 1)$ the set of all μ-measurable functions $x(g)$ of G with finite

$$\| x(g) \|_p = \left\{ \int_G | x(g) |^p \mu(dg) \right\}^{\frac{1}{p}} .$$

For arbitrary $x(g) \in L^1(G)$, $y(g) \in L^p(G)$ and

$$z(g) = x \times y(g) = \int_G x(h) y(h^{-1}g) \mu(dh) , \tag{1}$$

we have

$$\| z \|_p = \| x \times y \|_p \leqq \| x \|_1 \| y \|_p . \tag{2}$$

Defining the multiplication by (1) and putting

$$\| x \| = \text{Max.} (\| x \|_1, \| x \|_p) , \tag{3}$$

the intersection $L^{(1,p)}(G)$ of $L^1(G)$ and $L^p(G)$ thus becomes a non-commutative normed ring[1]. But, generally speaking, $L^{(1,p)}(G)$ has not a unit element. Adjoining therefore formally the unit e, I. E. Segal considered the set of all

$$\mathfrak{z} = \lambda e + x(g) ; \quad \lambda = \text{complex number}, \quad x(g) \in L^{(1,p)}(G) ,$$

and called it the group ring $R^{(1,p)}(G)$ of G[2]. But we would rather prefer to call $L^{(1,p)}(G)$ itself the group ring of G. We shall give in this paper certain close relations between G and $L^{(1,p)}(G)$, some of which are generalizations of the results of I. E. Segal.

§ 2. We consider representations of G and $L^{(1,p)}(G)$, i. e. homomorphic mappings of G and $L^{(1,p)}(G)$ into matrices, whose components are complex numbers[3].

Our main theorem is then:

Theorem 1. There is a one-to-one correspondence between continuous[4] representations of $L^{(1,p)}(G)$ and bounded continuous representations of G in the following sense:

i) For a given continuous representation $x(g) \rightarrow T(x)$ of $L^{(1,p)}(G)$, there corresponds uniquely a bounded continuous representation $a \rightarrow D(a)$ of G, so that it holds

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1) For normed rings cf. I. Gelfand: Normierte Ringe, Rec. Math., **51** (1941), 37–58.

2) I. E. Segal: The group ring of a locally compact group, I, Proc. Nat. Acad. Sci., U. S. A. **27** (1940).

3) For the representation of G, we do not require that the unit of G corresponds to the unit matrix.

4) The topology in $L^{(1,p)}(G)$ is of course given by the norm $\| x \|$ in (3).