PAPERS COMMUNICATED

12. Projective Parameters in Projective and Conformal Geometries.

By Kentaro YANO.

Mathematical Institute, Tokyo Imperial University.

(Comm. by S. KAKEYA, M.I.A., Feb. 12, 1944.)

§ 1. *Projective parameters in projective geometry.*

In an n-dimensional space A_n with the affine connection Γ^i_{jk}, a system of curves called paths is defined by the differential equations of the form

$$(1.1)\qquad \frac{d^2x^i}{ds^2}+\Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}=0 \qquad (i,j,k,\dots=1,2,\dots,n)$$

as autoparallel curves, where s is called affine parameter on each path. Conversely, if we are given the differential equations of the form (1.1) in an n-dimensional space X_n, we can define a symmetric affine connection in this space taking Γ^i_{jk} as the components of the connection. The study of the properties of these differential equations constitutes the affine geometry of paths[1]. But, an affine connection is not defined uniquely by the system of paths (1.1). H. Weyl[2] and L. P. Eisenhart[3] have independently shown that any two affine connections whose components $\bar{\Gamma}^i_{jk}$ and Γ^i_{jk} are related by the equations of the form

$$(1.2)\qquad \bar{\Gamma}^i_{jk}=\Gamma^i_{jk}+\delta^i_j\psi_k+\delta^i_k\psi_j\,,$$

where ψ_j are components of an arbitrary covariant vector not necessarily gradient, give the same paths. In this sense, the change over from $\bar{\Gamma}^i_{jk}$ to Γ^i_{jk} is called the projective change of affine connections, and the study of those properties which are invariant under such changes of affine connections is called the projective geometry of paths[4].

To study the projective geometry of paths, T. Y. Thomas[5] has introduced the functions

$$(1.3)\qquad \Pi^i_{jk}=\Gamma^i_{jk}-\frac{1}{n+1}(\delta^i_j\Gamma^a_{ak}+\delta^i_k\Gamma^a_{aj})\,,$$

which are invariant under projective change of affine connections (1.2).

1) L. P. Eisenhart and O. Veblen: The Riemann geometry and its generalisation. Proc. Nat. Acad. Sci. **8** (1922), pp. 19–23.

2) H. Weyl: Zur Infinitesimalgeometrie: Einordnung der projektiven und der konformen Auffassung. Göttinger Nachrichten (1921), pp. 99–112.

3) L. P. Eisenhart: Spaces with corresponding paths. Proc. Nat. Acad. Sci. **8** (1922), pp. 233–238.

4) O. Veblen: Projective and affine geometry of paths. ibidem, pp. 347–350.

5) T. Y. Thomas: On the projective and equi-projective geometries of paths. ibidem, **11** (1925), pp. 199–203.