41. *Normed Rings and Spectral Theorems, IV**.

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1. *Introduction.* The spectral theorem given in the third note[1] constitutes, in the special case when $T(M)$ is an enumerably-valued step function, a refinement with an error estimation of E. Schmidt's procedure of the approximate calculation of the greatest proper value of the integral equation. The result is then similar to that due to Temple and Collatz[2]. The purpose of the present note is to show that our treatment may be extended to obtain an approximate calculation of the lower proper values.

At this juncture, I intend to correct the misprints in III: 1) the right hand side of (iv) on page 72, line 5 must be read as

$$\frac{1}{\sqrt{\tau(\mu)}}\sqrt{\frac{F(T^{2n})}{F(T^{2(n+1)})}-\frac{F(T^{2n+1})^2}{F(T^{2(n+1)})^2}} \qquad (\text{as} \quad n\to\infty)$$

2) $\lim_{n\to\infty}\tau_{n+1}(\mu-\varepsilon)=1$ on page 73, line 6 must be read as $\lim_{n\to\infty}\tau_{n+1}(\mu-\varepsilon)=\tau(\mu-0)=\tau(\mu)$.

2. *The Theorem.* As in III, let $\boldsymbol{R}$ be the totality of the real-valued continuous functions $S(M)$ on a bicompact Hausdorff space $\mathfrak{M}$, and let $F(S)$ be a positive linear functional on $\boldsymbol{R}$ such that $F(I)=1$ where $I(M)\equiv 1$. Then

$$F(T)=\int_{\mathfrak{M}}T(M)\varphi(dM)=\int_{\lambda_0}^{\lambda_1}\lambda d\tau(\lambda),\ \lambda_0=\inf_M T(M),\ \lambda_1=\sup_M T(M),$$
$$\tau(\lambda)=\varphi\big(M;\ T(M)<\lambda\big).$$

We assume that $\lambda_0>0$ and that $\tau(\lambda)$ be of the form

$$(\text{A})\quad\begin{cases}\tau(\mu^{(2)}+0)>\tau(\mu^{(2)}-0) & \tau(\lambda)=\text{constant for } \mu^{(2)}<\lambda<\mu^{(1)}\\ \tau(\mu^{(1)}+0)>\tau(\mu^{(1)}-0), & \tau(\lambda)=\text{constant for } \mu^{(1)}<\lambda<\mu^{(0)}\\ \tau(\mu^{(0)}+0)>\tau(\mu^{(0)}-0), & \tau(\lambda)=\tau(\mu^{(0)}+0) \text{ for } \lambda>\mu^{(0)}\end{cases}$$

$\mu^{(0)}$, $\mu^{(1)}$ may respectively be called as *the maximal, the next maximal spectrum of F referring to T.*

We put

$$\frac{F(T^{2n+1})}{F(T^{2(n+1)})}=\frac{1}{\mu_n^{(0)}}\qquad\sqrt{\frac{F(T^{2n})}{F(T^{2(n+1)})}}=\frac{1}{\nu_n^{(0)}},$$

* The cost of this research has been defrayed from the Scientific Research Expenditure of the Department of Education.

1) Proc. **20** (1944), 71. This note will be referred to as III.

2) G. Temple: Proc. London Math. Soc., **29** (1929), 257. L. Collatz: Math. Zeitschr., **46** (1940), 692. Our formula (iv) ((iv)′) does not contain the unknown value $\mu^{(1)}(\mu^{(2)})$ explicitely. Moreover in (iv)″ the values $\mu^{(0)}$, $\mu^{(1)}$, $\mu^{(2)}$ are only implicitely needed. These are the main difference of our results from Temple-Collatz's.