56. *Normed Rings and Spectral Theorems, V.*

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1. *Introduction.* Recently, M. Krein[1] published a generalisation of the Plancherel's theorem to the case of locally compact (=bicompact) abelian group. The result is much important, since it reveals the hitherto hidden algebraic character of the classical Fourier analysis. However, Krein's proof of the positivity of the functional J is somewhat complicated and moreover it seems that his paper lacks the proof of 3° which is the key to the proof of the positivity of J. The purpose of the present note is to show that a complete proof may be obtained by making use of the preceding note[2]. It is also to be remarked that the theorem below constitutes an extension of 3° and that, by virtue of this extension, Krein's arguments may be much simplified.

2. *A theorem of positivity.* Let G be a locally compact, separable abelian group and let X be the group (without topology for the moment) of continuous characters $\chi(g)$ of G. Then, by Haar's invariant measure dg, we may define the linear space $L_p(G)$ $(\infty > p \geqq 1)$ of complex-valued measurable functions $x(g)$ such that $|x(g)|^p$ is summable over G:

$$(1) \qquad \|x\|_p = \sqrt[p]{\int |x(g)|^p \, dg} < \infty .$$

A multiplication $x * y$ is introduced in $L_1(G)$ by the convolution:

$$(2) \qquad x * y(g) = \int x(g-h) y(h) dh .$$

By adjoining formally[3] a unit e to $L_1(G)$ we obtain a normed ring $R(G)$ by the norm $\|z\|$ and the multiplication $*$:

$$(3) \qquad \begin{cases} z = \lambda e + x(g), \quad \|z\| = |\lambda| + \|x\| \quad (\lambda = \text{complex number}), \\ z_1 = \lambda_1 e + x_1(g), \quad z_2 = \lambda_2 e + x_2(g), \\ z_1 * z_2 = \lambda_1\lambda_2 e + \lambda_1 x_2(g) + \lambda_2 x_1(g) + x_1 * x_2(g). \end{cases}$$

Such ring is considered by *I.* Gelfand and *D.* Raikov[4].

We next introduce a new normed ring to be denoted as $\bar{R}_{op}(G)$. For any $x \in L_1(G)$ and for any $y \in L_2(G)$ we have

$$(4) \qquad x * y(g) \in L_2(G), \qquad \|x * y\|_2 \leqq \|x\|_1 \cdot \|y\|_2 ,$$

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* The cost of this research has been defrayed from the Scientific Research Expenditure of the Department of Education.

1) C. R. URSS, **30** (1941), No. 6.
2) Proc. **19** (1943), p. 356. This note will be referred to as (I).
3) The trivial case of the discrete group G is excluded in the following lines.
4) C. R. URSS, **28** (1940), No. 3.