72. *On the Torse-forming Directions in Riemannian Spaces.*

By Kentaro YANO.
Mathematical Institute, Tokyo Imperial University.
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§ 0. It is well known that a vector $v^\lambda(s)$ defined on each point of the curve $x^\lambda(s)$ in a Riemannian space V_n is said to be parallel along the curve, if it satisfies the differential equations of the form

$$(0.1) \qquad \frac{\delta v^\lambda}{ds} \equiv \frac{dv^\lambda}{ds} + \{{}^{\lambda}_{\mu\nu}\} v^\mu \frac{dx^\nu}{ds} = 0 ,$$

$\{{}^{\lambda}_{\mu\nu}\}$ being the Christoffel symbols of the second kind. Following E. Cartan, the Euclidean connection without torsion of the Riemannian space being defined by

$$dM = dx^\lambda e_\lambda , \qquad de_\mu = \{{}^{\lambda}_{\mu\nu}\} dx^\nu e_\lambda ,$$

if we develop the curve on the tangent space at a point of the curve, the directions $v^\lambda(s)$ defined as above along the curve will be found to be parallel along the curve developed on the tangent space, for, the equations (0.1) just show that the geometrical variation of the vector $v^\lambda e_\lambda$ along the curve vanishes. This will be the most natural interpretation of Levi-Civita's parallelism.

On the other hand, we have studied, in a previous paper[1)], the concurrency of the directions defined along a curve in Riemannian spaces. A vector $v^\lambda(s)$ defined on each point of the curve $x^\lambda(s)$ in a Riemannian space is said to be concurrent along the curve, if it satisfies the differential equations of the form

$$(0.2) \qquad \frac{dx^\lambda}{ds} + \frac{\delta \alpha v^\lambda}{ds} = 0 ,$$

where α is a suitable function of s. In fact, these equations show that the geometrical variation of the point $M + \alpha v^\lambda e_\lambda$ vanishes along the curve, and hence, if we develop the curve $x^\lambda(s)$ on the tangent space at a point of the curve, all the directions $v^\lambda(s)$ defined on each point of the curve pass through the fixed point $M + \alpha v^\lambda e_\lambda$.

Generalizing these concepts of parallelism and concurrency, we shall study in the present Note the torse-forming directions in Riemannian spaces. The torse-forming directions may be considered in affinely or projectively connected spaces. We have already indicated an application of torse-forming directions to the geometrical interpretation of the projective transformations of asymmetric affine connections[2)].

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1) K. Yano: Sur le parallélisme et la concourance dans l'espace de Riemann. Proc. **19** (1943), 189–197.

2) K. Yano: Über eine geometrische Deutung der projektiven Transformationen nicht-symmetrischer affiner Übertragungen. Proc. **20** (1944), 284–287.