96. Relations between Measure and Topology in some Boolean Space.

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Let Ω be a bicompact Hausdorff space the closure of whose open set is open. We assume that the class $\mathfrak{C}$ of all the closed-open sets constitutes the base of Ω. $\mathfrak{C}$ is a finitely additive class which contains Ω and the empty set $\mathfrak{O}$. Let there be defined on $\mathfrak{C}$ a Jordan measure $m(E)$ with the following two conditions:

1 $m(\Omega)=1$, $m(E)=0$ if and only if $E=\mathfrak{O}$.

2 $\lim_{n\to\infty} m(E_n)=m\Big((\bigcup_{n=1}^{\infty} E_n)^a\Big)$ for any ascending sequence $\{E_n\}$ of sets $\in \mathfrak{C}$[1].

The purpose of the present note is to discuss the relations between measure and topology in Ω. Our main result is resumed in the theorems 10, 11 and 13 below.

Theorem 1. We have

$$\sum_{n=1}^{\infty} m(E_n) \geqq m\Big((\bigcup_{n=1}^{\infty} E_n)^a\Big)$$

for every sequence $\{E_n\}$ of sets $\in\mathfrak{C}$, and the equality holds good if and only if E_n are mutually disjoint. In particular, we have

$$\sum_{n=1}^{\infty} m(E_n) = m(\bigcup_{n=1}^{\infty} E_n)$$

if $\bigcup_{n=1}^{\infty} E_n \in \mathfrak{C}$. Thus the Jordan measure $m(E)$ is countably additive on C.

Definition 1. (of outer measure m^*). For any set $A \leqq \Omega$, $m^*(A)$ denotes the infimum of $m(E)$ where $E\in\mathfrak{C}$, $E \geqq A$

Theorem 2.

(i) $m^*(A) \leqq m^*(B)$ if $A \leqq B$

(ii) $m^*(A) = m(A)$ if $A\in\mathfrak{C}$

(iii) $m^*(A+B) \leqq m^*(A)+m^*(B)$

(iv) $m^*(A) = m^*(A^a)$

Definition 2. (of inner measure m_*). For any set $A \leqq \Omega$, $m_*(A)$ denotes the supremum of $m(E)$ where $E\in\mathfrak{C}$, $E \leqq A$.

Theorem 3.

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1) A^a, A^c and A^i respectively denote the closure, the complement and the interior of A.