116. *On the Diophantine Analysis of Algebraic Functions.*

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Tue's and Siegel's theorems are very important and significant in the recent development of the diophantine analysis. We see that the greater part of these theories will hold good even if algebraic functions are considered, instead of algebraic numbers. The purpose of this paper is to treat the outline of this analogy. Details will be discussed in another paper.

We consider an algebraic function $\omega(t)$ with elements of a corpus Ω as coefficients. The Laurent's expansion of $\omega(t)$ at infinity is of the form

$$\omega(t)=\sum_{n-p}^{-\infty} a_n t^n = a_p t^p + a_{p-1}t^{p-1} \cdots + a_0 + \frac{a_{-1}}{t} + \frac{a_{-2}}{t^2} + \cdots .$$

$$a_p \neq 0 .$$

Now we take a fixed real number $e > 1$ and put

$$\varphi\big(\omega(t)\big) = e^p \quad \text{for} \quad \omega(t) \neq 0 , \quad \varphi(0)=0 .$$

Then φ gives an evaluation (Bewertung) of the corpus of algebraic functions $\omega(t)$, such that

$$\varphi\big(w_1(t).\, \omega_2(t)\big) = \varphi\big(\omega_1(t)\big).\, \varphi\big(\omega_2(t)\big) , \tag{1}$$

$$\varphi\big(\omega_1(t)+\omega_2(t)\big) \leqq \max\Big(\varphi\big(\omega_1(t)\big),\, \varphi\big(\omega_2(t)\big)\Big) . \tag{2}$$

In (2) the inequality happens eventually in the case $\varphi\big(\omega_1(t)\big) = \varphi\big(\omega_2(t)\big)$. If we denote the integral part $a_p t^p + a_{p-1}t^{p-1} + \cdots + a_0$ of $\omega(t)$ with the notation $[\omega(t)]$, we can define the continued fraction $[A_0(t), A_1(t), A_2(t), \ldots]$ of an algebraic function $\omega(t)$, putting

$$A_0(t) = [\omega(t)] , \qquad \omega(t) = A_0(t) + \frac{1}{\omega_1(t)} ,$$

$$[\omega_1(t)] = A_1(t) , \qquad \omega_1(t) = A_1(t) + \frac{1}{\omega_2(t)} ,$$

$$[\omega_2(t)] = A_2(t), \ldots .$$

Then the fundamental theory of continued fraction can be applied without modification.

Theorem 1. (Lagrange). The necessary and sufficient condition that the continued fraction of $\omega(t)$ be recurrent is as follows:

1° $\omega(t)$ satisfies an irreducible quàdratic equation with integral coefficients

$$a(t)\omega^2 + b(t)\omega + c(t) = 0 ;$$