127. On the Osculating Representation for a Dynamical System with Slow Variation.

Second Note.

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In a preceding note[1] the author has enunciated one of the results of his study on the osculating representation for a dynamical system with slow variation, in which the associated curtailed system of differential equations for the motion of the dynamical system is osculatingly represented by quasi-periodic functions of Bohl's class. In the present note I give one of the results under the additional condition that the Hamiltonian function H is, besides being analytic with regard to the first pairs of variables x_i and y_i in a domain $|x_i|, |y_i| < D$, $(i=1,2,\dots,m)$, and periodic in t with period 2π as in the preceding note, also analytic with regard to the second pairs of variables ξ_j and η_j in a domain $|\xi_j - A_j|, |\eta_j - B_j| < \varDelta$ in the immediate neighbourhood of the initial point $\xi_j = A_j$, $\eta_j = B_j$, $(j=1,2,\dots,n)$, where A_j and B_j are constants, in anticipating the possibility of attacking the problems as to the foundations of the theory of long period variations in celestial mechanics and of the theories of degenerate systems and of adiabatic invariants in quantum mechanics.

The differential equations of the problem have been reduced in the preceding note[3] to the normalised form

$$
(1)\quad \left\{
\begin{aligned}
&\frac{d\bar{x}_i}{dt} = \left\{-\sqrt{-1}\,\lambda_i + \left(\frac{\partial K^{(s)}}{\partial c_i}\right)\right\}\cdot \bar{x}_i, \\
&\frac{d\bar{y}_i}{dt} = -\left\{-\sqrt{-1}\,\lambda_i + \left(\frac{\partial K^{(s)}}{\partial c_i}\right)\right\}\cdot \bar{y}_i, \quad (i=1,2,\dots,m), \\
&\frac{d\xi_j}{dt} = \frac{\sqrt{-1}}{2}\left(\frac{\partial K^{(s)}}{\partial \eta_j}\right), \quad \frac{d\eta_j}{dt} = -\frac{\sqrt{-1}}{2}\left(\frac{\partial K^{(s)}}{\partial \xi_j}\right), \\
&\qquad\qquad (j=1,2,\dots,n),
\end{aligned}
\right.
$$

in which $K^{(s)}$ is a finite power series arranged in ascending powers *of the constants $c_1, c_2, \dots, c_m$, beginning with the terms of the second* degree, the coefficients of the various powers of c_i 's being in the present case analytic with respect to ξ_j, η_j and t in the immediate neighbourhood of $\xi_j = A_j$, $\eta_j = B_j$, $(j=1,2,\dots,n)$, and for all values of t, and is periodic in t with period 2π.

By the change of variables

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1) Y. Hagihara, Proc. **20** (1944), 617.

2) A part of the results has been communicated to the American Mathematical Society in December 1928. *Cf.*, Bull. Amer. Math. Soc., **35** (1929), 178.

3) Y. Hagihara, *loc. cit.*, Equation (6).