PAPERS COMMUNICATED

126. On the Osculating Representation for a Dynamical System with Slow Variation.

First Note.

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In a preceding note[1] the present author has given a theorem concerning the dynamical systems with slow variation and obtained the maximum time interval in which a semi-convergent representation of the solution deviates by less than a given amount from the true solution of the differential equations for the dynamical system in question. In the present note I have the privilege to report one of the results I have been able to reach in the case when the differential equations can be approximated, not necessarily convergently, by a quasi-periodic function of Bohl's class, as the general integrals usually adopted for the solution in the planetary theory, although not uniformly convergent as has been proved by Poincaré, are taken to be such a class of functions[2]. The dynamical system under consideration is meant for a simplification of the planetary and satellite systems existing in nature. I intend in future to extend the research towards the theory of the general form of the integrals and the stability of the planetary motion in general.

Consider a system of differential equations

$$(1)\quad \begin{cases} \dfrac{dx_i}{dt}=\dfrac{\partial H}{\partial y_i}, & \dfrac{dy_i}{dt}=-\dfrac{\partial H}{\partial x_i}, \qquad (i=1,2,\ldots,m), \\ \dfrac{d\xi_j}{dt}=\dfrac{\partial H}{\partial \eta_j}, & \dfrac{d\eta_j}{dt}=-\dfrac{\partial H}{\partial \xi_j}, \qquad (j=1,2,\ldots,n), \end{cases}$$

where H is a function of $2m+2n+1$ variables x_i, y_i, ξ_j, η_j, $(i=1,2,\ldots,m;\ j=1,2,\ldots,n)$, and t, and, together with its partial derivatives of the first and the second orders with respect to x_i, y_i, ξ_j and η_j, is Lipschitzian with regard to ξ_j and η_j, and is analytic with regard to x_i, y_i and t for all values of ξ_j, η_j and t and for all values of x_i and y_i in a domain

$$(2)\qquad |x_i|,\ |y_i|<D, \qquad (i=1,2,\ldots,m),$$

with a finite positive constant D, and is periodic in t with period 2π. Assume that we have a solution $x_i=y_i=0$, $\xi_j=A_j$, $\eta_j=B_j$, $(i=1,2,\ldots,m;$

1) Y. Hagihara, Proc. **7** (1931), 44.

2) See, for example, Delaunay, Théorie du Mouvement de la Lune. Mém. Acad. Sc. Inst. Imp. France. **27** (1860), **29** (1867); Newcomb, Journ. de Math. pure et appl., [ii] **16** (1871), 321; Smithsonian Contr. to Knowledge, 1874, 281; Lindstedt, Ann. École Norm. Sup., [iii] **1** (1884), 85; Bohlin, Bihang til Svenska Vet. Acad. Handlingar. **14** (1888); Poincaré, Les Méthodes Nouvelles de la Mécanique Céleste. **2** (1893).