141. On Student's Test.

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1. *Introduction*[1].

Let (Ω, P_θ) be *the population of a sample* with the parameter $\theta \in \Theta$. Let H be any subset of Θ. By the *hypothesis* H, $H \leqq \Theta$, we understand the hypothesis "$\theta \in H$". If H consists of only one point, it is called a simple hypothesis, and if H contains at least two points, it is called a composite hypothesis. When $\theta_1 \in \Theta - H$, the simple hypothesis θ_1 is called an *alternative hypothesis of* H.

Let K be any subset of Ω. The *test of the hypothesis* H *by the critical region* K is defined as the following rule of rejection:

(1) if the *realized sample point* ω belongs to K, H is rejected, and
(2) if $\omega \in K$, H is non-rejected.

With regard to this test we may consider two types of error. The *error of the first type* e_{I} is that which is made by rejecting H when it is true, while the *error of the second type* e_{II} is that which is made by accepting H when it is false.

e_{I} is measured by the probability $P_\theta(K)$, $\theta \in H$, say $e_{\text{I}}(K, \theta)$.

Definition 1. K is called to be *regular*, if $e_{\text{I}}(K, \theta)$, $\theta \in H$, is independent of θ, as far as θ runs over H. The common value is called the *size* of K and is denoted by $e_{\text{I}}(K)$.

e_{II} is measured by the probability $P_{\theta_1}(\Omega - K)$, $\theta_1 \in \Theta - H$, say $e_{\text{II}}(K, \theta_1)$.

Definition 2. K_0 is called to be the *most powerful against* an assigned alternative θ_1, if K_0 is regular and if, whenever K is any regular critical region with the same size as K_0, we have $e_{\text{II}}(K_0, \theta_1) \leqq e_{\text{II}}(K, \theta_1)$.

The main purpose of this note is to prove that Student's method of testing the hypothesis concerning the mean of normal populations is the best one in a certain sense (Theorems 4 and 6). For the proof we make use of Theorems 3 and 5 concerning the regularity of critical regions, which are the immediate results from Theorem 1 and 2 due to Mr. K. Yosida, to whom the author owes much in this research and wishes to express his hearty thanks.

2. *Theorems of Mr. K. Yosida.*

For the later use we shall prove two theorems due to Mr. K. Yosida.

Theorem 1. Let $f(x)$, $-\infty < x < \infty$, be any real-valued bounded measurable function, and $g(x)$ be any function $\in L_1(-\infty, \infty)$ whose Fourier transform does not vanish on the real axis. Then the condition that the integral

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1) Cf. Wilks: The Theory of Statistical Inference, Princeton (1937), chap. V.