Correction to “Strong topological transitivity of some classes of operators”

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Some of the results in [1] are based on part (i) of [2, Theorem 5], restated as Proposition 1.4 in [1]. The aim of this note is to point out that its proof, as presented in [2] is wrong. At the moment it is an open question whether part (i) of [2, Theorem 5] is true or not, and this has consequences for the results in [1]. The statement made in [1, Remark 1.5] is based on it, and it is not known whether it is true or false. The proof of [1, Prop. 2.12] is based on [1, Prop. 1.4]; this result is true, we present an alternative proof below.

The mistake made in the proof of part (i) of [2, Theorem 5] is the following. Suppose that $\mathcal{H}$ is a Hilbert space and $T : \mathcal{H} \to \mathcal{H}$ is an invertible bounded linear operator. In the proof of [2, Theorem 5] it is stated that “M is a T-invariant closed set if and only if $M^\perp$ is a T^*-invariant closed set”. This is true, but there is no guarantee that $M^\perp$ is nontrivial whenever M is nontrivial; we can claim that $M^\perp \neq \mathcal{H}$ but we cannot claim that $M^\perp \neq \{0\}$.

Observe that [1, Prop. 1.4] is now an open question. In fact, we do not know whether an invertible strongly topologically transitive operator on $\mathcal{H}$ exists. By [2, Prop. 6] this is equivalent to the existence of a hypertransitive operator on $\mathcal{H}$, and this is equivalent to the existence of an operator on $\mathcal{H}$ without nontrivial closed invariant subsets, which is still an open question.

We finally present a direct proof of [1, Prop. 2.12]. Suppose that M_ϕ^* is an invertible strongly topologically transitive operator on H^2. Then $M_{1/\phi}^* = (M_\phi^*)^{-1}$ is hypertransitive, by [2, Prop. 6]. This cannot be true since every reproducing

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