A Note on Article 36 in Gauss's *Disquisitiones*

A Ramificated Story in the Margin of the Re-Writing of Section II

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Even 150 years after Carl Friedrich Gauss's death, his magnum opus, the *Disquisitiones Arithmeticae* ([5], I = DA), has lost nothing of its fascination. Gauss and his work have been described as the occurence of a comet in a clear sky, inaugurating a new era of mathematics, of number theory in particular. The *Disquisitiones*, however, did not fall from the sky, and many links with not only previous number-theoretical works but also with near forgotten mathematicians as Hindenburg or with a tradition of German textbooks can be found (see [1], part 3). This opens up the often dense if not cryptic text of the *Disquisitiones* and helps to gain a richer understanding of the environment in which Gauss's treatise was written und thus of its proper innovations. This article will illustrate this point by focusing on a small example, DA's article 36, showing its rooting in current discussions in German mathematics before 1800.

In the DA, Article 36 follows Gauss's treatment of the Chinese Remainder Problem and considers the system (I) $[A, B, C$ being relatively prime]:

$$\begin{aligned} X &\equiv a \pmod{A} \\ X &\equiv b \pmod{B} \\ X &\equiv c \pmod{C} \end{aligned}$$

Solving this system of congruences for $a = 1, b = 0, c = 0$; then for $a = 0, b = 1, c = 0$ and for $a = 0, b = 0, c = 1$ generates the 3 solutions for X: α, β, γ. The formula to calculate all solutions to (I) for a given triple (a, b, c) is: $X \equiv \alpha a + \beta b + \gamma c$ (mod. ABC).

And then follows an example, one of the few instances of an everyday problem in the DA, the calculation of year's number in a Julian period:

> This formula is useful for the chronological problem, where the Julian year is asked, when given the indiction [a], golden number [b] and solar cycle [c]. Here $A = 15$,