Equivariant quantizations and Cartan connections

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1 Introduction

In the framework of geometric quantization, it is common to define a quantization procedure as a linear bijection from the space of classical observables to a space of differential operators acting on wave functions (see [31]).

In our setting, the space of observables (also called the space of *Symbols*) is the space of smooth functions on the cotangent bundle T^*M of a manifold M, that are polynomial along the fibres. The space of differential operators $\mathcal{D}_{\frac{1}{2}}(M)$ is made of differential operators acting on half-densities.

It is known that there is no natural quantization procedure : the spaces of symbols and of differential operators are not isomorphic as representations of $\mathrm{Diff}(M)$.

The idea of G-equivariant quantization is to reduce the set of (local) diffeomorphisms under consideration. If a Lie group G acts on M by local diffeomorphisms, this action can be lifted to symbols and to differential operators. A G-equivariant quantization was defined by P. Lecomte and V. Ovsienko in [21] as a G-module isomorphism from symbols to differential operators.

They first considered the projective group $PGL(m+1,\mathbb{R})$ acting locally on the manifold $M=\mathbb{R}^m$ by linear fractional transformations and defined the notion of *projectively equivariant quantization.* They proved the existence of such a quantization and its uniqueness, up to some natural normalization condition.

In [11], the authors considered the group $SO(p+1,q+1)$ acting on the space $\mathbb{R}^{p+q}$ or on a manifold endowed with a flat conformal structure. There again, the result was the existence and uniqueness of a *conformally equivariant quantization.*

1991 *Mathematics Subject Classification :* 17B66, 53B05, 53B10, 53C10, 81R05.

Key words and phrases : Equivariant quantizations, projective structures, Cartan connections, invariant differentiation.