Noncommutative Algebraic Geometry: from pi-algebras to quantum groups

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Abstract

The main purpose of this paper is to provide a survey of different notions of algebraic geometry, which one may associate to an arbitrary noncommutative ring R. In the first part, we will mainly deal with the prime spectrum of R, endowed both with the Zariski topology and the stable topology. In the second part we focus on quantum groups and, in particular, on schematic algebras and show how a noncommutative site may be associated to the latter. In the last part, we concentrate on regular algebras, and present a rather complete up to date overview of their main properties.

Introduction.

The main purpose of this paper is to present a survey of the subject commonly known as "noncommutative algebraic geometry". The first two sections treat the prime spectrum of a noncommutative ring, endowed with its canonical structure sheaf. This approach is useful for algebras with enough prime ideals, like algebras satisfying a polynomial identity (pi-algebras for short). Allowing for a more general topology (induced by Artin-Rees ideals) provides a geometry for rings with the so-called second layer condition. However, many interesting algebras fall outside the scope of these techniques. They arise naturally in the study of quantum groups, an introduction to which is given in section 3.

The last two sections are devoted to projective noncommutative geometry. The central object of study here is the quotient category $Proj$, which, for a commutative algebra, is equivalent to the category of quasi-coherent sheaves on its projective variety. There is a large class of graded algebras, the so-called schematic algebras,

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