On Veldkamp Lines

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Abstract

One says that Veldkamp lines exist for a point-line geometry Γ if, for any three distinct (geometric) hyperplanes A, B and C (i) A is not properly contained in B and (ii) $A \cap B \subseteq C$ implies $A \subset C$ or $A \cap B = A \cap C$. Under this condition, the set $\mathcal{V}$ of all hyperplanes of Γ acquires the structure of a linear space – the *Veldkamp space* – with intersections of distinct hyperplanes playing the role of lines. It is shown here that an interesting class of strong parapolar spaces (which includes both the half-spin geometries and the Grassmannians) possess Veldkamp lines. Combined with other results on hyperplanes and embeddings, this implies that for most of these parapolar spaces, the corresponding Veldkamp spaces are projective spaces.

The arguments incorporate a model of partial matroids based on intersections of sets.

1 Introduction

Let Γ be a point-line geometry, that is, a rank two incidence system $(\mathcal{P}, \mathcal{L})$ with each object incident with at least two others. The objects of $\mathcal{P}$ are called "points"; those of $\mathcal{L}$ are called "lines"; nothing is assumed by this nomenclature. A *subspace of* Γ is a subset S of $\mathcal{P}$ such that any line L with two of its incident points in S has all its incident points in S. We assume without any real loss that distinct lines possess distinct sets of incident points (distinct *point-shadows*) and so may themselves be regarded as subsets of $\mathcal{P}$.

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