Equivariant weak n-equivalences

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The notion of n-type was introduced by J.H.C. Whitehead ([22, 23]) where its clear geometric meaning was presented. Following J.L. Hernandez and T. Porter ([12, 13]) we use the term *weak n-equivalence* for a map $f : X \to Y$ of path-connected spaces which induces isomorphisms $\pi_k(f) : \pi_k(X) \to \pi_k(Y)$ on homotopy groups for $k \leq n$. Certainly, weak n-equivalence of a map determines its n-connectedness but not conversely. For J.H. Baues ([2, page 364]) n-types denote the category of spaces X with $\pi_k(X) = 0$ for $k > n$. The n-type of a CW-space X is represented by P_nX, the n-th term in the Postnikov decomposition of X. Then the n-th Postnikov section $p_n : X \to P_nX$ is a weak n-equivalence. Much work has been done to classify the n-types and find equivalent conditions for a map $f : X \to Y$ to be a weak n-equivalence. J.L. Hernandez and T. Porter ([12]) showed how with this notion of weak n-equivalence and with a suitable notion of n-fibration and n-cofibration one obtains a Quillen model category structure ([20]) on the category of spaces. The case of weak n-equivalences mod a class $\mathcal{C}$ of groups (in the sense of Serre) was analyzed by C. Biasi and the second author ([3]). E. Dror ([5]) pointed out that weak equivalences of certain spaces (including nilpotent and complete spaces) can be described by means of homology groups. Then in 1977 J.H. Baues ([1]) proved the Dual Whitehead Theorem for maps of $\mathfrak{R}$-Postnikov spaces (of order $k \geq 1$), where $\mathfrak{R}$ is a commutative ring.

Given the growing interest in equivariant homotopy, it is not surprising that notions of equivariant n-types have been studied. For instance algebraic models for equivariant 2-types have been presented by I. Moerdijk and J.-A. Svensson ([18])

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