Elliptic spaces with the rational homotopy type of spheres

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1 Introduction

This paper is directed towards an understanding of those p-elliptic spaces which have the rational homotopy type of a sphere, by classifying the algebraic models which occur when the space satisfies an additional 'large prime' hypothesis, relative to the prime p. The main results of the paper are given at the end of this section.

Definition 1.1 [10] A topological space Z is p-elliptic if it has the p-local homotopy type of a finite, 1-connected CW complex and the loop space homology $H_*(\Omega Z; \mathbf{F}_p)$, with coefficients in the prime field of characteristic p, is an elliptic Hopf algebra. (That is: finitely-generated as an algebra and nilpotent as a Hopf algebra [9]). ■

The Milnor-Moore theorem shows that the $\mathbf{Q}$-elliptic spaces are precisely those spaces which have the rational homotopy type of finite, 1-connected CW complexes and have finite total rational homotopy rank. This class of spaces is important because of the *dichotomy theorem* (the subject of the book [8]) which states that a finite, 1-connected complex either has finite total rational homotopy rank or the rational homotopy groups have exponential growth when regarded as a graded vector space. Moreover, elliptic spaces are the subject of the Moore conjectures, asserting that the homotopy groups of a finite, 1-connected CW complex have finite exponent at all primes if and only if it is $\mathbf{Q}$-elliptic.

The p-elliptic spaces form a sub-class of the class of $\mathbf{Q}$-elliptic spaces. A p-elliptic space Z is known to satisfy the following important properties [10, 11].

1. $H^*(Z; \mathbf{F}_p)$ is a Poincaré duality algebra.

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