Classification of Surfaces in $\mathbb{R}^3$ which are centroaffine-minimal and equiaffine-minimal

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Abstract

We classify all surfaces which are both, centroaffine-minimal and equiaffine-minimal in $\mathbb{R}^3$.

1 Introduction.

In equiaffine differential geometry, the variational problem for the equiaffine area integral leads to the equiaffine minimal surfaces, such surfaces have zero equiaffine mean curvature $H(e) = 0$. These surfaces were called affine minimal by Blaschke and his school ([1]). Calabi [2] pointed out that, for locally strongly convex surfaces with $H(e) = 0$, the second variation of the area integral is negative, so he suggested that the surfaces with $H(e) = 0$ should be called affine maximal surfaces. Wang [13] studied the variation of the centroaffine area integral and introduced the centroaffine minimal hypersurfaces, such hypersurfaces have the property that $\mathrm{trace}_G \widehat{\nabla}\widehat{T} \equiv 0$, where G is the centroaffine metric, $\widehat{\nabla}$ the centroaffine metric connection and $\widehat{T}$ the centroaffine Tchebychev form (see the definitions in §2). The study of Wang [13] leads to the more general definitions (and the generalizations) of the Tchebychev operator and Tchebychev hypersurfaces, see [5], [8], [9] and [10].

In this paper, we consider the centroaffine surfaces which are centroaffine-minimal and equiaffine-minimal in $\mathbb{R}^3$. We give the following classification theorem.

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