A structure sheaf on the projective spectrum of a graded fully bounded noetherian ring

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Abstract

In this note, we show how abstract localization and graded versions of the Artin-Rees property may be applied to construct structure sheaves over the projective spectrum $Proj(R)$ of a graded fully bounded noetherian ring R.

Introduction.

The main purpose of this note is to construct structure sheaves on the projective spectrum $Proj(R)$ of a graded fully bounded noetherian ring R, generalizing the analogous construction in the commutative case. The principal tool to realize this will be abstract localization (in the sense of [5, 6, 7, 16, 19, et al]) in the category $R-\mathbf{gr}$ of graded left R-modules.

Actually, if I is a graded ideal contained in $R_+ = \bigoplus_{n>0} R_n$, then for any graded left R-module M, one associates to the Zariski open subset $D_+(I) \subseteq Proj(R)$ the graded R-module of quotients $Q_I^g(M)$, obtained by localizing M with respect to the torsion at positive powers I^n of I. In general, however, this construction only yields a separated presheaf on $Proj(R)$, which is not necessarily a sheaf. Of course, one may derive from this a sheaf on $Proj(R)$, by applying the usual sheafification process. Nevertheless, this is highly unsatisfactory in the present context, since passing over to the associated sheaf enlarges the modules of global sections, which, in particular, makes the sheaf thus obtained rather useless for any representational aims, for example.

If we want to obtain a sheaf directly, we may either restrict the class of graded R-modules, we wish to represent or change the topology on $Proj(R)$ in a reasonable

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