The quantifier complexity of NF

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Abstract

Various issues concerning the quantifier complextity (i.e., number of alternations of like quantifiers) of Quine's theory NF, its axiomatizations, and some of its subtheories, are discussed.

1 Introduction

In this paper I shall investigate various issues concerning the quantifier complexity of NF. The main motivation for this work concerns the consistency problem for NF (see for example Boffa [1977]) and is as follows.

1. Kaye [1991] proved a generalization of a theorem of Specker's [1962] concerning the equiconsistency of NF and an extension of the theory of simple types, TST, the so-called *ambiguity axioms* Amb(ϕ). In particular, the author's modification shows Specker's theorem holds even if these ambiguity axioms are restricted to ϕ in a certain complexity class. The exact class of formulas here depends on the on the complexity of axiomatizations of NF. Roughly speaking, if we can find axiomatizations of NF of low complexity, the modification of Specker's theorem is more powerful and potentially more useful. Obtaining upper bounds on the complexity of axiomatizations of NF is the subject of section 1 below.

2. There is a great deal of interest in determining fragments of NF that are actually decidable. Hinnion [1972] has shown that (assuming NF to be consistent) the set of universal consequences of NF is decidable and complete in a certain sense. (See also Kreinovič and Oswald [1982].) Forster believes that a similar result for the stratified $\forall_2$ consequences of NF should hold, and although he has the best part of a

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