Scaled trace forms of central simple algebras

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Introduction

Any central simple algebra over a field of characteristic not two has a well-defined non-singular quadratic form called the trace form attached to it. This quadratic form was studied in [6] from the viewpoint of the algebraic theory of quadratic forms. In this article we examine a generalization of trace forms to "scaled trace forms" which are defined via the reduced trace map together with scaling by a non-zero element of the algebra.

In section 1 of the paper we give our basic definitions and obtain necessary and sufficient conditions for the scaled trace forms to be non-singular. In section 2 we deal with the "split" case, i.e. the case when the algebra is a full matrix algebra over the field. In section 3 we investigate the algebraic invariants of scaled trace forms. We show that the determinant of a scaled trace form is, up to sign, equal to the reduced norm of the scaling element. Also we give formulae for the signature at each ordering when the underlying field is formally real. We cannot calculate the Hasse-Witt invariant in general but we determine it in two special cases only. In sections 4 and 5 we relate scaled trace forms to the transfer homomorphism of quadratic form theory and use them to obtain information about the kernel and image of the extension of scalars homomorphism of Witt rings.

We use the terminology and notation of the book of Scharlau [9] and refer the reader to this book for any further background information on quadratic form theory.

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