The prescribed mean curvature equation for a revolution surface with Dirichlet condition

A. L. Maestripieri M. C. Mariani

Abstract

We give conditions on a continuous and bounded function H in R^2 to obtain at least two weak solutions of the mean curvature equation with Dirichlet condition for revolution surfaces with boundary, using variational methods.

Introduction

The prescribed mean curvature equation with Dirichlet condition for a vector function $X : B \longrightarrow R^3$ is the system of non linear partial equations

$$(1) \begin{cases} \triangle X = 2H(X)X_u \wedge X_v & in \quad B \\ X = X_0 & in \quad \partial B \end{cases}$$

where B is the unit disk in R^2, $\wedge$ denotes the exterior product in R^3 and $H : R^3 \longrightarrow R$ is a given continuous function.

When H is bounded and X_0 is in the Sobolev space $H^1(B, R^3)$, we call $X \in H^1(B, R^3)$ a weak solution of (1) if $X \in X_0 + H_0^1(B, R^3)$ and for every $\phi \in C_0^1(B, R^3)$

$$\int_B \nabla X \cdot \nabla \phi + 2H(X)X_u \wedge X_v \cdot \phi = 0.$$

In certain cases, weak solutions are obtained as critical points in $X_0 + H_0^1(B, R^3)$ of the functional

$$D_H(X) = D(X) + 2V(X)$$

Received by the editors June 1995.
Communicated by J. Mawhin.
1991 *Mathematics Subject Classification :* Primary 35 – Secondary 35J60.
Key words and phrases : Mean curvature, Dirichlet problem, Revolution surface.