An equation $\dot{z} = z^2 + p(t)$ with no 2π-periodic solutions

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Abstract

The Mawhin conjecture - that there exists a 2π-periodic $p : \mathbf{R} \to \mathbf{C}$ such that $\dot{z} = z^2 + p(t)$ has no 2π-periodic solutions - is confirmed by the use of Fourier expansions.

In 1992 R.Srzednicki [4], [5] proved that for any 2π-periodic continuous $p : \mathbf{R} \to \mathbf{C}$ the equation $\dot{z} = \overline{z}^2 + p(t)$ has a 2π-periodic solution. J.Mawhin [3] conjectured that the similarly looking problem $\dot{z} = z^2 + p(t)$ could have no 2π-periodic solutions for some p. The first example of such p was constructed by J.Campos and R.Ortega [1]. This work was intended as an attempt to provide with another example by the use of a quite different method. During the preparation of this paper J.Campos [2] determined all the possible dynamics of this equation and found other examples.

Conjecture 1 *There exists $R_0 \in [1,2]$ such that the equation*

$$\dot{z} = z^2 + Re^{it} \tag{1}$$

has no 2π-periodic solutions for $R = R_0$.

Let us define the sequence

$$a_1 = 1 , \quad a_n = \frac{1}{n}\sum_{k=1}^{n-1} a_k a_{n-k}. \tag{2}$$

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