The non-Archimedian Laplace Transform

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Abstract

Topological properties of the spaces of analytic test functions and distributions are investigated in the framework of the general theory of non-archimedean locally convex spaces. The Laplace transform, topological isomorphism, is introduced and applied to the differential equations of non-archimedean mathematical physics (Klein-Gordon and Dirac propagators).

Introduction.

Last years a number of quantum models over non-archimedean fields was proposed (quantum mechanics, field and string theory, see for example books [1,2] and references in these books). As usual, new physical formalisms generate new mathematical problems. In particular, a lot of differential equations with partial derivatives were introduced in connection with non-archimedean mathematical physics (Schrödinger, Heisenberg, Klein- Gordon,...), see [1,2]. In the ordinary real and complex analysis, one of the most powerful tools to investigate equations with constant coefficients are the Fourier and Laplace transforms. It is not a simple problem to introduce these transforms in the non- archimedean case, see [3 - 7]. There is a number of different approaches and the main problem is always that the Fourier and Laplace transforms are not isomorphisms. There exist non-zero functions with zero Fourier or Laplace transform. A new approach to this problem was proposed in [2, 8] on the basis of the non-archimedean theory of analytical distributions.

In this paper we study the properties of the non-archimedean locally convex spaces of distributions and test functions. The main result in this direction is that

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