Collineations of Subiaco and Cherowitzo hyperovals

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Abstract

A Subiaco hyperoval in $\mathrm{PG}(2, 2^h)$, $h \geq 4$, is known to be stabilised by a group of collineations induced by a subgroup of the automorphism group of the associated Subiaco generalised quadrangle. In this paper, we show that this induced group is the full collineation stabiliser in the case $h \not\equiv 2 \pmod 4$; a result that is already known for $h \equiv 2 \pmod 4$. In addition, we consider a set of $2^h + 2$ points in $\mathrm{PG}(2, 2^h)$, where $h \geq 5$ is odd, which is a Cherowitzo hyperoval for $h \leq 15$ and which is conjectured to form a hyperoval for all such h. We show that a collineation fixing this set of points and one of the points $(0, 1, 0)$ or $(0, 0, 1)$ must be an automorphic collineation.

1 Introduction

In the Desarguesian projective plane $\mathrm{PG}(2, q)$ of even order $q = 2^h$, $h \geq 1$, an *oval* is a set of $q + 1$ points, no three collinear, and a *hyperoval* is a set of $q + 2$ points no three of which are collinear. A hyperoval $\mathcal{H}$ can be written, with a suitable choice of homogeneous coordinates for $\mathrm{PG}(2, q)$, as

$$\mathcal{H} = \mathcal{D}(f) = \{(1, t, f(t)) \colon t \in \mathrm{GF}(q)\} \cup \{(0, 1, 0), (0, 0, 1)\}$$

for some function f on $\mathrm{GF}(q)$ satisfying $f(0) = 0$ and $f(1) = 1$, see [6, 8.4.2]. (Note that in [6] an oval is called a $(q + 1)$-arc and a hyperoval is called an oval.)

We are interested in calculating the stabiliser in the automorphism group of $\mathrm{PG}(2, q)$ of some recently discovered hyperovals. The automorphism group of

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