A class of Buekenhout unitals in the Hall plane

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Abstract

Let U be the classical unital in $PG(2,q^2)$ secant to ℓ_∞. By deriving $PG(2,q^2)$ with respect to a derivation set disjoint from U we obtain a new unital U' in the Hall plane of order q^2. We show that this unital contains O'Nan configurations and is not isomorphic to the known unitals of the Hall plane, hence it forms a new class of unitals in the Hall plane.

1 Introduction

A **unital** is a 2-$(q^3+1, q+1, 1)$ design. A unital embedded in a projective plane of order q^2 is a set U of q^3+1 points such that every line of the plane meets U in 1 or $q+1$ points. A line is a **tangent line** or a **secant line** of U if it contains 1 or $q+1$ points of U respectively. A point of U lies on 1 tangent and q^2 secant lines of U. A point not in U lies on $q+1$ tangent lines and q^2-q secant lines of U.

An example of a unital in $PG(2,q^2)$, the Desarguesian projective plane of order q^2, is the **classical unital** which consists of the absolute points and non-absolute lines of a unitary polarity. It is well known that the classical unital contains no **O'Nan configurations**, a configuration of four distinct lines meeting in six distinct points (a quadrilateral). In 1976 Buekenhout [4] proved the existence of unitals in all translation planes of dimension at most 2 over their kernel.

Let U be the classical unital in $PG(2,q^2)$ secant to ℓ_∞. We derive $PG(2,q^2)$ with respect to a derivation set disjoint from U. Let U' be the set of points of $\mathcal{H}(q^2)$,

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