Universal Properties of the Corrado Segre Embedding

Corrado Zanella

Abstract

Let $\mathcal{S}(\Pi_0, \Pi_1)$ be the *product* of the projective spaces Π_0 and Π_1, i.e. the semilinear space whose point set is the product of the point sets of Π_0 and Π_1, and whose lines are all products of the kind $\{P_0\} \times g_1$ or $g_0 \times \{P_1\}$, where P_0, P_1 are points and g_0, g_1 are lines. An *embedding* $\chi : \mathcal{S}(\Pi_0, \Pi_1) \to \Pi'$ is an injective mapping which maps the lines of $\mathcal{S}(\Pi_0, \Pi_1)$ onto (whole) lines of Π'. The classical embedding is the Segre embedding, $\gamma_0 : \mathcal{S}(\Pi_0, \Pi_1) \to \overline{\Pi}$. For each embedding χ, there exist an automorphism α of $\mathcal{S}(\Pi_0, \Pi_1)$ and a linear morphism $\psi : \overline{\Pi} \to \Pi'$ (i.e. a composition of a projection with a collineation) such that $\chi = \alpha\gamma_0\psi$. (Here $\alpha\gamma_0\psi$ maps P onto $\psi(\gamma_0(\alpha(P))) =: P\alpha\gamma_0\psi$.) As a consequence, every $\mathcal{S}(\Pi_0, \Pi_1)$ which is embedded in a projective space is, up to projections, a Segre variety.

1 Introduction

Most classical varieties represent as points of a projective space some geometric objects. So such varieties are (*projective*) *embeddings*, which are somewhat canonical. For instance, take an h-flat ${}^hU$ (i.e. a subspace of dimension h) of an n-dimensional projective space Π over a commutative field F. ${}^hU$ can be associated with $\binom{n+1}{h+1}$ coordinates, the so-called *Plücker coordinates*, or *Grassmann coordinates*. They are defined up to a factor. So ${}^hU$ can be represented as a point of a $(\binom{n+1}{h+1} - 1)$-dimensional projective space. We call *Plücker map* this representation. The image of

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