Octonion hermitian quadrangles

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Abstract

We introduce hermitian generalized quadrangles over the octonions. These quadrangles extend the classical hermitian quadrangles over the reals, the complex numbers and the quaternions in a natural way. For the smallest quadrangle, $H_3\mathbb{O}$, we show that the group Spin(9) acts as a line-transitive automorphism group.

Introduction

Octonions or Cayley division algebras are complex and beautiful objects which are, unfortunately, absent in finite geometry. They can be used to construct a family of particularly nice generalized quadrangles. The smallest of these quadrangles, $H_3\mathbb{O}$, has a line-transitive automorphism group. These quadrangles generalize and extend in a natural way the classical standard hermitian quadrangles over the reals, the complex numbers, or the quaternions. They where first described by Ferus-Karcher-Münzner [1] in connection with Clifford algebras and isoparametric hypersurfaces; later, Thorbergsson [7] proved by a topological argument that they are quadrangles.

We here take a different approach to these quadrangles: instead of real Clifford algebras we use the octonions, and we give an algebraic proof that the geometries are quadrangles. The approach via Clifford algebras can be found in [2]. It should be said that although the quadrangles originate from differential and topological geometry, the whole construction is purely algebraic and works whenever the field $\mathbb{R}$ of real numbers is replaced by a real closed field.

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