Spectral semi-norm of a p-adic Banach algebra

Alain Escassut Nicolas Mainetti

Abstract

Let K be a complete ultrametric algebraically closed field, with respect to a non trivial absolute value, and let A be a commutative K-Banach algebra with identity. Let $Mult(A, \| \, . \, \|)$ be the set of continuous multiplicative semi-norms of K-algebra (with respect to the norm $\| \, . \, \|$ of A) and let $Mult_m(A, \| \, . \, \|)$ the set of the $\varphi \in Mult(A, \| \, . \, \|)$ whose kernel is a maximal ideal of A. If the norm of A is equal to its spectral semi-norm $\| \, . \, \|_{si}$ defined as $\|x\|_{si} = \lim\limits_{n \to +\infty} \| \, x^n \, \|^{\frac{1}{n}}$, we prove that $\|t\|_{si} = \sup\{\psi(t) | \ \psi \in Mult_m(A, \| \, . \, \|)\}$, without any additional condition on K. Moreover, if A has no divisors of zero, denoting by $s(x)$ the spectrum of any $x \in A$, we have $\|t\|_{si} = \sup\{|\lambda| \mid \lambda \in s(x)\}$. If $\sup\{|\lambda| \mid \lambda \in s(t)\} = \|t\|_{si}$ for every $t \in A$, then $s(t)$ is infraconnected for all $t \in A$ if and only if A has no non trivial idempotents. In particular, this applies when A has no divisors of zero. In $Mult(A, \| \, . \, \|)$ we define pseudo-dense sets, and show that a subset Σ of $Mult(A, \| \, . \, \|)$ containing $Mult_m(A, \| \, . \, \|)$ is pseudo-dense if and only if for all $t \in A$ we have $\|t\|_{si} = \sup\{\psi(t) | \ \psi \in \Sigma\}$.

1 Introduction and results

Let L be a complete ultrametric field, and let K be a complete ultrametric algebraically closed field with respect to a non trivial absolute value. L is said to be *strongly valued* if its residue class field, or if its valuation group, is not countable. As usual, given $a \in K$, $r > 0$, we put $d(a, r) = \{x \in K \mid |x - a| \leq r\}$, $d(a, r^-) = \{x \in K \mid |x - a| < r\}$, $C(a, r) = \{x \in K \mid |x - a| = r\}$.
Besides, given $s > r$, we put $\Gamma(a, r, s) = d(a, s^-) \setminus d(a, r)$.

Received by the editors October 1996.
Communicated by R. Delanghe.
1991 *Mathematics Subject Classification* : 46S10, 12J25.
Key words and phrases : p-adic analysis, spectral norm, ultrametric Banach algebra.