Classification of Riemannian 3-manifolds with distinct constant principal Ricci curvatures *

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Abstract

We prove that the local isometry classes of Riemannian 3-manifolds with distinct constant Ricci eigenvalues are parametrized by three arbitrary functions of two variables. This improves essentially the earlier result by A.Spiro and F.Tricerri from [9].

1 Introduction

The problem of how many Riemannian metrics exist on the open domains of $\boldsymbol{R}^3$ with prescribed constant Ricci eigenvalues $\rho_1 = \rho_2 \neq \rho_3$ was completely solved in the series of papers [3],[2] and [7]. The main existence theorem says that the local isometry classes of these metrics are always parametrized by *two arbitrary functions of one variable*. Some nontrivial explicit examples are presented in [3], as well.

The case of distinct constant Ricci eigenvalues is more interesting. Here the first nontrivial examples have been presented by K.Yamato [10], and some others in [4]. Finally, in [5], nontrivial explicit examples have been constructed for *every choice* of the Ricci eigenvalues $\rho_1 > \rho_2 > \rho_3$. (All examples in [10] are *complete* Riemannian manifolds but the range of the admissible triplets of Ricci eigenvalues is restricted by certain algebraic inequalities. Outside this range it seems that the corresponding metrics must always be incomplete.) In [6] an *explicit classification* was done under some additional geometric conditions.

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