Finite Dimensional Hopf Algebras Coacting on Coalgebras

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Introduction

Let H be a finite dimensional Hopf algebra and let C be a left H-comodule coalgebra. In [2], a Morita-Takeuchi context arising from a left H-comodule coalgebra has been constructed. Utilizing that Morita- Takeuchi context we may characterize the Hopf-Galois coactions on coalgebras, and use it to prove the duality theorem for crossed coproducts. In this note, we show that the Morita-Takeuchi context constructed in [2] is generated by the left comodule ${}_{C\rtimes H}C$, where $C \rtimes H$ is the smash coproduct coalgebra of C by H. As a consquence, we obtain that the coaction of Hopf algebra H on C is Galois if and only if ${}_{C\rtimes H}C$ is a cogenerator. This dualizes the corresponding result in [1]. Another functorial description of Galois coactions is in Theorem 2.8, which is the dualization of the weak structure theorem in [4].

In Section 3, we define the cotrace map for an H^*-coextension C/R. There are various descriptions of the cotrace map being injective. For instance, the comodule ${}_{C\rtimes H}C$ is an injective comodule; the canonical map G in the Morita-Takeuchi context is injective; the cohom functor $h_{C\rtimes H-}(C,-)$ is equivalent to the cotensor functor $C\square_{C\rtimes H}-$ cf.Theorem 3.5.

1 Preliminaries

Throughout k is a fixed field. All coalgebras, algebras, vector spaces and unadorned $\otimes$, Hom, etc, are over k. C, D always denote coalgebras and H is a Hopf algebra. We refer to [9] for detail on coalgebras and comodules. We adapt the usual sigma notation for the comultiplications of coalgebras, and adapt the following sigma notation

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