LS-category of classifying spaces

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ABSTRACT. — Let X be a 1-connected topological space of finite type. Denote by $B aut X$ the base of the universal fibration of fibre X. We show in this paper that the Lusternik-Schnirelmann category of $B\,aut\,X$ is not finite when X is a wedge of spheres or a finite CW-complex with finite dimensional rational homotopy.

Introduction

In this paper X will denote a simply connected CW-complex of finite type. Recall that the Lusternik-Schnirelmann category of a topological space, $cat(S)$, is the least integer n such that S can be covered by $(n+1)$ open subsets contractible in S, and is ∞ if no such n exists.

Denote by X_0 the localization of X at zero, the rational Lusternik-Schnirelmann category, $cat_0(X)$, is defined by $cat_0(X) = cat(X_0)$. This invariant satisfies

$$nil\, H^*(X,\, \mathbb{Q}) \, \leq e_0(X) \leq cat_0(X), \quad ([6], [12]), \tag{i}$$

$$cat_0(X) \leq cat(X) \ ([12]), \tag{ii}$$

where $nil\, H^*(X, \mathbb{Q})$ is the nilpotence of the cohomology ring with rational coefficients and $e_0(X)$ the Toomer invariant ([12]).

In this text we will use the theory of minimal models. The Sullivan minimal model of X is a free commutative cochain algebra $(\Lambda Z, d)$ such that $dZ \subset \Lambda^{\geq 2} Z$. Moreover $Z^n \cong Hom_{\mathbb{Q}}(\pi_*(X) \otimes \mathbb{Q},\, \mathbb{Q})$ ([10], [5]).

The Quillen minimal model of X is a free chain Lie algebra $(\mathbb{L}(V),\, \delta)$ satisfying $\delta V \subset \mathbb{L}^{\geq 2} V$ and the graded vector space V is related to the cohomology of X by

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