Topology and closed characteristics of K-contact manifolds.

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Abstract

We prove that the characteristic flow of a K-contact form has at least n+1 closed leaves on a closed 2n+1-dimensional manifold. We also show that the first Betti number of a closed sasakian manifold with finitely many closed characteristics is zero.

1 Preliminaries

A contact form on a 2n+1-dimensional manifold M is a 1-form α such that the identity

$$\alpha \wedge (d\alpha)^n \neq 0$$

hold everywhere on M. Given such a 1-form α, there is always a unique vector field ξ satisfying $\alpha(\xi) = 1$ and $i_\xi d\alpha = 0$. The vector field ξ is called the *characteristic vector field* of the contact manifold (M, α) and the corresponding 1-dimensional foliation is called a contact flow.

The 2n-dimensional distribution $D(x) = \{v \in T_xM/\alpha(x)(v) = 0\}$ is called the *contact distribution*. It carries a 1-1 tensor field J such that $J^2 = -I_{2n}$, where I_{2n} is the identity 2n by 2n matrix. The tensor field J extends to all of TM by requiring $J\xi = 0$.

Also, the contact manifold (M, α) carries a nonunique riemannian metric g adapted to α and J in the sense that the following identities are satisfied

$$d\alpha(X, Y) = g(X, JY)$$

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