Measurability of linear operators in the Skorokhod topology

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Abstract

It is proved that bounded linear operators on Banach spaces of "cadlag" functions are measurable with respect to the Borel σ-algebra associated with the Skorokhod topology.

1 Introduction and notation.

Throughout this paper $\mathbb{C}^n$ is understood to be equipped with an inner product $\langle \cdot, \cdot \rangle$, defined by

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i \overline{y}_i$$

for all $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in $\mathbb{C}^n$. We shall write $|x| = \sqrt{\langle x, x \rangle}$ for all $x \in \mathbb{C}^n$.
A function $f : [0,1] \to \mathbb{C}^n$ is said to be a *cadlag function* ("continu à droite, limite à gauche") if for all $t \in [0,1]$ one has:

$$\lim_{s \downarrow t} f(s) = f(t+) = f(t) \qquad \text{and} \qquad \lim_{s \uparrow t} f(s) = f(t-) \quad \text{exists}$$

As can be proved in an elementary way, for every cadlag function f and every $\varepsilon > 0$ the set

$$\{t \in [0,1] : |f(t) - f(t-)| \geq \varepsilon\}$$

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