Mutations in finite groups

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1 Introduction

One generally studies the different types of algebraic structures from the equivalence relation "being isomorphic", thus establishing a series of invariants and canonical models for each equivalence class under the mentioned relation. The main goal of this paper is to study how we can minimally quantify, by means of certain parameters, the degree of non-isomorphy between two given groups of the same order, i.e., to study the degree of invariance between two distinct equivalence classes under the relation "being isomorphic" in the category of finite groups.

The search of an efficient estimator of the qualification "non-isomorphic groups", which allow us to know if they are "hardly or nearly" isomorphic, has led us to define the concept of mutation, which, formalized in the category of internal Ω-algebras, is widely studied in this paper in the category of finite groups.

We present the concept and the results relative to mutations, according with the three stages their study has taken us chronologically. First of all, from the comparative analysis of pairs of similar structures having underlying sets of the same cardinality, i.e., groups of the same small order, 4,6,8, etc., it arises the concept of mutation, as being a bijection maximally satisfying the homomorphy condition, in order to minimize the number of times one has to mutate the group law to get an isomorphism from the given map.

It is also worth noting that the concept of mutation furthermore lets us rapidly check whether two given distinct presentation correspond to the same group.

Once introduced the concept of mutation between non-isomorphic groups and dealt with its properties we obtain in a second stage the evolutive chains in the sets

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