Transfinite methods in geometry

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Dedicated to J. A. Thas on his fiftieth birthday

Abstract

Some very difficult questions in finite geometry (for example, concerning ovoids, spreads, and extensions) have fairly trivial resolutions in the infinite case, by standard transfinite induction. We give some old and new results of this kind, and a number of open questions. The paper is expository in nature, and is intended to illustrate the technique rather than give the most general applications.

1 Introduction

Jef Thas has contributed to almost every part of finite geometry. Among these are the related topics of ovoids, spreads and extensions. To cite just a couple among many important papers, we mention [12], [13]. It is our purpose here to show that, for infinite geometries, Jef's great ingenuity is not needed; constructions are easy, and everyone can have a go.

The basic tool is transfinite induction, which is outlined briefly in Section 2. In the following sections, we construct extensions of Steiner systems and projective planes; blocking sets, ovoids, and parallelisms of projective spaces; and partitions into ovoids for a variety of quadrangle-like structures, leading to flat geometries with diagrams like $C_2.C_2$ and "large sets" of Steiner systems.

The paper also contains a number of open questions.

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